


Adaptive multi-dimensional Taylor network tracking control for a class of switched nonlinear systems with input nonlinearity

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Abstract

In this paper, a multi-dimensional Taylor network (MTN)-based adaptive tracking control approach is proposed for a class of switched nonlinear systems with input nonlinearity. Firstly, the input nonlinearity is assumed to be bounded by a sector interval. Secondly, with the help of MTNs approximating the unknown nonlinear functions, a novel adaptive MTN control scheme has the advantages of low cost, simple structure and real time feature is developed via backstepping technique. It is shown that the tracking error finally converges to a small domain around the origin and all signals in the closed-loop system are bounded. Finally, two examples are given to demonstrate the effectiveness of the proposed control scheme.

Keywords

Multi-dimensional Taylor network, tracking control, switched nonlinear systems, input nonlinearity

Introduction

As a special class of hybrid systems, switched systems have attracted much attention since they exist in many physical and engineering systems, such as mechanized operation (Chai et al., 2016), one-link manipulator (Niu et al., 2018). Therefore, the investigation on control design strategies for switched systems has attracted more and more attention (Gao et al., 2011; Kundu et al., 2016; Liu et al., 2015; Niu et al., 2017). During scores of years developments, many interesting results have been reported (Johansson and Rantzer, 1998; Morse, 1996, 1997; Willem, 2003). For example, for a class of switched nonlinear systems, Xu and Antsaklis (2004) developed a novel control strategy to deal with the optimal control problems. Kosmatopoulos and Ioannou (2002) proposed a novel robust adaptive control scheme for a class of multi-input nonlinear systems with nonlinearly dynamics. For a class of switched time-varying delays uncertain linear systems, Li et al. (2008) proposed a robust tracking control approach. Vu et al. (2007) studied the stability problem of switched nonlinear systems under average dwell-time (ADT) switching signals. In fact, many switched nonlinear systems fail to preserve stability under arbitrary switching signals. Therefore, the investigation of the controller design and stability analysis of switched nonlinear systems is very necessary.

On the other hand, many approximation-based adaptive control schemes, such as multi-dimensional Taylor network (MTN) (Han, 2018a; Han et al., 2018; Han and Yan, 2018a; Kang and Yan, 2018; Yan and Kang, 2017), neural networks (NNs) (Han, 2018b; Han et al., 2019a; Wang and Huang,

2005; Wang et al., 2013b; Yin et al., 2017) and fuzzy logic systems (FLSs) (Chen et al., 2009; Zhou et al., 2011), have been found to be particularly useful for the control of nonlinear systems. Among the existing research results, MTN-backstepping-based adaptive control schemes have been found to be particularly suitable for solving the control problems of nonlinear systems. MTN is a three-layer network, including input layer (information input layer), middle layer (information processing layer, which is composed of polynomials) and output layer (information output layer). At first, MTNs were used to solve the problems of the identification and forecasting of nonlinear system (Zhou and Yan, 2014a, 2014b). Meanwhile, due to the traits of simple structure, less calculation, capacity of study, and self adapting, MTNs are suitable for the control of nonlinear systems. For example, based on discrete MTN, Yan and Kang (2017) studied the problem of asymptotic tracking and dynamic regulation for a class of

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single input single output (SISO) nonlinear systems. Kang and Yan (2018) discussed the problem of stability analysis and dynamic regulation for a class of SISO nonlinear systems with time-varying delay. For a class of nonlinear systems with unknown control direction, Han et al. (2019b) proposed a MTN-based adaptive tracking control approach. Han (2018a), Han and Yan (2018a, 2018b), Yan and Han (2019) addressed the problems of adaptive tracking control for several classes of stochastic nonlinear systems using MTN control approach and backstepping technique. However, most of the results not considered a special class of hybrid systems, which are switched nonlinear systems. Recently, a growing attention has been paid on switched nonlinear systems due to the fact that, in the real world, many practical engineering systems can be described as switched nonlinear systems (Han et al., 2009; Li et al., 2018; Li et al., 2017; Zhao et al., 2015). Therefore, studying the problems of adaptive approximation-based controller design and stability analysis of closed-loop systems for switched nonlinear systems is of essential importance.

Through the above analysis, it is not difficult to find that the mentioned above literature only focus on the control strategy design for switched nonlinear systems without non-smooth nonlinear characteristics, such as dead-zone, backlash and saturation. However, input nonlinearities characteristics frequently exist in practical control systems, and many significant developments have been achieved; for example, Wang et al. (2004) proposed robust adaptive control architecture for a class of nonlinear systems preceded by a dead-zone. Ibrir et al. (2007) developed a robust adaptive compensation algorithm for a class of nonlinear linearizable uncertain systems with non-symmetric dead-zone input. Wang et al. (2013a) discussed the problem of adaptive fuzzy tracking control for a class of pure-feedback stochastic nonlinear systems with input saturation. Model-based adaptive control was investigated in Chen et al. (2011) for a class of uncertain MIMO nonlinear systems with input constraints. Chiang et al. (2007) presented a novel sliding mode control scheme for uncertain unified chaotic systems with input nonlinearity. Xiang and Chen (2011) were concerned with robust stabilization of chaotic systems with mismatched perturbations and input nonlinearities. Another important issue encountered in practice is continuous nonlinearly input (Wu et al., 2012), and many interesting results have been reported (Chiang et al., 2007; Kebriaei and Yazdanpanah, 2010). However, to the best of the authors' knowledge, there are few results available on MTN-based adaptive control for switched nonlinear systems with input nonlinearity, which motivates our research.

Motivated by the above observations, it can be concluded that it is necessary to investigate the adaptive tracking control problem for a class of switched nonlinear systems with input nonlinearity. In the controller design, MTNs are used to evaluate unknown functions, and then a novel MTN-based adaptive tracking control scheme is proposed via backstepping technique. Compared with the existing literatures, the main advantages of this paper are as follows:

- (1) The approximation-based adaptive MTN method can solve the tracking control problems of the switched

nonlinear systems with input nonlinearity. In addition, the proposed method can deal with tracking problems, not just stabilization problems. Also, two goals have been achieved, that are: (i) The tracking error finally converges to a small domain around the origin; (ii) All signals of the closed-loop system are bounded.

- (2) Although MTN-based adaptive control approaches for nonlinear systems (Han, 2018b; Han et al., 2019b) and stochastic nonlinear systems (Han, 2018a; Han and Yan, 2018a, 2018b) have been developed, they only investigated the nonlinear systems without considering the case of input constraint. In the work of Han et al. (2018), the authors investigated the problem of adaptive tracking control for a class of stochastic nonlinear systems with input dead-zone using MTN. However, the mentioned above control schemes cannot solve the tracking control problems of switched nonlinear systems with input nonlinearity.
- (3) The switched nonlinear systems studied in this paper are subject to external disturbance, and the control directions are unknown, which means the structure of systems are more general than some existing systems, such as those in Ibrir et al. (2007) and Yau and Yan (2007).
- (4) On the one hand, in order to reduce the number of MTNs, in each step of the backstepping, we lump all unknown functions into a suitable unknown function that is approximated by only one MTN. Meanwhile, the MTN-based control scheme proposed in this paper has a good real-time performance thanks to the simple structure of MTN. Therefore, the computation burden is reduced.

The remaining of this paper are organized as follows. Section 2 presents some preliminaries. The MTN-based adaptive tracking controller design procedure and stability analysis are presented in Section 3. Two examples are provided in Section 4 to illustrate the effectiveness of the proposed controller design scheme. Finally, some conclusions are drawn in Section 5.

System description and preliminaries

Notations: \mathbb{R} denotes the set of all real numbers, \mathbb{R}^i indicates the real i -dimensional space. For a given vector or matrix X , X^T stands for its transpose and $\text{Tr}(X)$ denotes its trace when X is square. $\theta^T P_{m_i}(s)$ denotes a MTN has i inputs and the highest power of the polynomial of middle layer is m_i , and θ is the weight vector of MTN.

Problem description

Consider the following switched nonlinear systems with unknown control directions and input nonlinearity

$$\begin{cases} \dot{x}_i = g_{i,\sigma(t)}(\bar{x}_i)x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i) + d_{i,\sigma(t)} \\ 1 \leq i \leq n-1 \\ \dot{x}_n = \phi(u) + f_{n,\sigma(t)}(\bar{x}_n) + d_{n,\sigma(t)} \\ y = x_1 \end{cases} \quad (1)$$

where $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the state vector with $\bar{x}_i = [x_1, \dots, x_i]^T \in \mathbb{R}^i$, $i = 1, \dots, n$. $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and the system output, respectively. $\sigma(t) : [0, \infty) \rightarrow M = \{1, 2, \dots, m\}$ stands for a piecewise continuous switching signal, and $\sigma(t) = j, j \in M$ implies that the j -th subsystem is active. $f_{i, \sigma(t)} : \mathbb{R}^i \rightarrow \mathbb{R}$ ($i = 1, \dots, n$) and $g_{i, \sigma(t)} : \mathbb{R}^i \rightarrow \mathbb{R}$ ($i = 1, \dots, n-1$) are unknown smooth nonlinear functions, and $d_{i, \sigma(t)}$ ($i = 1, \dots, n$) is external disturbance.

Remark 2.1: Different from Li et al. (2019) and Wang et al. (2018), in our paper, the external disturbance $d_{i, \sigma(t)}$ dependent on switching signal $\sigma(t)$, which means that different subsystems of switched nonlinear system are subject to different external disturbances. Thus, we may conclude that the considered switched nonlinear systems in this paper are more general.

In system (1), $\phi(u)$ is a continuous nonlinear function with $\phi(0) = 0$ belonging to the sector $[\lambda_m, \lambda_M]$, that is to say, $\phi(u)$ satisfies the following mathematical expression (Kebriaei and Yazdanpanah, 2010)

$$\lambda_m u \leq \phi(u) \leq \lambda_M u, \quad u \geq 0 \quad (2)$$

$$\lambda_m u \geq \phi(u) \geq \lambda_M u, \quad u < 0 \quad (3)$$

Remark 2.2: Combining (2) and (3), the following inequality holds

$$|\phi(u)| \leq \max\{\lambda_m, \lambda_M\}|u| = \bar{\lambda}|u| \quad (4)$$

where $\bar{\lambda} = \max\{\lambda_m, \lambda_M\}$ is a known positive constant.

For system (1) with the $\phi(u)$ satisfies (2) and (3), the control objective of this paper is to design an adaptive MTN-based controller such that the system output y follows the given reference signal y_d , and all signals in the closed-loop system are bounded.

To facilitate control system design, the following assumptions are needed.

Assumption 2.1: (Zhao et al., 2015). The given reference signal y_d and its time derivatives up to the n -th order are continuous and bounded.

Assumption 2.2: (Wang et al., 2014). For $i = 1, 2, \dots, n-1$ and $j \in M$, the sign of function $g_{i, j}(\cdot)$ does not change. Without loss of generality, it is further assumed that there exist two known constants b_m and b_M , such that

$$0 < b_m \leq g_{i, j}(\bar{x}_i) \leq b_M < \infty \quad (5)$$

Assumption 2.3: For $i = 1, 2, \dots, n$ and $j \in M$, there exist constants $\bar{d}_{i, j}$, such that

$$|d_{i, j}| \leq \bar{d}_{i, j} \quad (6)$$

MTN

In this paper, the unknown smooth nonlinear function in the system will be approximated by MTN $\theta^T P_{m_n}(s)$ on a compact set Ω_s .

Lemma 2.1: (Han et al., 2019b). For a compact set Ω_s and a continuous function $f(s)$ defined on Ω_s , then, for $\forall \varepsilon > 0$, there exists a MTN, such that

$$f(s) = \theta^{*T} P_{m_n}(s) + \delta(s) \quad (7)$$

where $s = [s_1, \dots, s_n]^T \in \Omega_s \subset \mathbb{R}^n$, $P_{m_n}(s) = \underbrace{[s_1, \dots, s_n]}_{1 \text{ term}}$, $\underbrace{s_1^2, \dots, s_n^2}_{2 \text{ term}}, \dots, \underbrace{s_1^m, \dots, s_n^m}_{m \text{ term}}]^T \subset \mathbb{R}^l$ and $\delta(s)$ denotes the approximation error and satisfies $|\delta(s)| \leq \varepsilon$. The optimal weight vector $\theta^* = [\theta_1, \dots, \theta_l]^T$ is defined as

$$\theta^* := \arg \min_{\theta \in \mathbb{R}^l} \left\{ \sup_{z \in \Omega_s} |f(z) - \theta^T P_{m_n}(z)| \right\} \quad (8)$$

Remark 2.3: In comparison with the NN, the structure of the MTN is relatively simple because there are no neural units on the middle layer of it. Specifically, MTN contains only addition and multiplication due the fact that its middle layer consists of an array of polynomials. Therefore, MTN-based controller has the advantages of low complexity and less computation.

Main results

Adaptive MTN controller design

Before proceeding with the approximation-MTN-based adaptive controller design, define constant θ_i ($i = 1, \dots, n$) as

$$\theta_i = \max\{\|\theta_{i, k}\| : k \in M\}$$

where $\theta_{i, k}$ is the MTN's weight vector, and its value will be given later.

Remark 3.1: It is obvious that θ_i is an unknown constant for every $i = 1, \dots, n$, and let $\hat{\theta}_i = \theta_i - \tilde{\theta}_i$, where $\tilde{\theta}_i$ is the estimation of constant θ_i .

Step 1: Define new variables as follows

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_2 &= x_2 - \alpha_1 \end{aligned}$$

where α_1 is the intermediate virtual control signal to be designed later.

Consider the Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}_1^2 \quad (9)$$

From (9), it follows that

$$\dot{V}_1 = z_1 (g_{1, j} x_2 + \bar{f}_{1, j} + d_{1, j}) - z_1^2 - \tilde{\theta}_1 \dot{\tilde{\theta}}_1 \quad (10)$$

where $\bar{f}_{1, j} = f_{1, j} - \dot{y}_d + z_1$ is an unknown function.

Then, by virtue of Lemma 2.1, for any given constant $\varepsilon_{1, j} > 0$, there exists a MTN $\theta_{1, j}^T P_{m_1}(z_1)$, such that

$$\bar{f}_{1,j} = \theta_{1,j}^T P_{m_1}(\mathbf{z}_1) + \delta_{1,j}(\mathbf{z}_1), \quad |\delta_{1,j}(\mathbf{z}_1)| \leq \varepsilon_{1,j} \quad (11)$$

where $\mathbf{z}_1 = [z_1]^T$ is the input vector and $\delta_{1,j}(\mathbf{z}_1)$ is the estimate error.

From (10) and (11), it follows that

$$\begin{aligned} \dot{V}_1 \leq & g_{1,j} z_1 z_2 + g_{1,j} z_1 \alpha_1 + \frac{1}{2} \varepsilon_{1,j}^2 + \frac{1}{2} \bar{d}_{1,j}^2 \\ & + \frac{1}{2} s_1^2 + \frac{1}{2s_1^2} z_1^2 \theta_1 P_{m_1}^T P_{m_1} - \tilde{\theta}_1 \dot{\hat{\theta}}_1 \end{aligned} \quad (12)$$

Taking the intermediate control signal α_1 as

$$\alpha_1 = -\frac{1}{b_m} \left(r_1 z_1 + \frac{1}{2s_1^2} z_1 \hat{\theta}_1 P_{m_1}^T P_{m_1} \right) \quad (13)$$

where $r_1 > 0$ and $s_1 > 0$ are design parameters.

Then, we have

$$\begin{aligned} g_{1,j} z_1 \alpha_1 &= -\frac{r_1}{b_m} g_{1,j} z_1^2 - \frac{1}{2b_m s_1^2} g_{1,j} z_1^2 \hat{\theta}_1 P_{m_1}^T P_{m_1} \\ &\leq -r_1 z_1^2 - \frac{1}{2s_1^2} z_1^2 \hat{\theta}_1 P_{m_1}^T P_{m_1} \end{aligned} \quad (14)$$

Combining (12), (13) and (14), we have

$$\begin{aligned} \dot{V}_1 \leq & g_{1,j} z_1 z_2 - r_1 z_1^2 + \frac{1}{2} \varepsilon_{1,j}^2 + \frac{1}{2} \bar{d}_{1,j}^2 \\ & + \frac{1}{2} s_1^2 + \tilde{\theta}_1 \left(\frac{1}{2s_1^2} z_1^2 P_{m_1}^T P_{m_1} - \dot{\hat{\theta}}_1 \right) \end{aligned} \quad (15)$$

Step i ($2 \leq i \leq n-1$): Define new variables

$$\begin{aligned} z_i &= x_i - \alpha_{i-1} \\ z_{i+1} &= x_{i+1} - \alpha_i \end{aligned}$$

where α_i is the intermediate virtual control signal to be designed later.

Consider the Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}_i^2 \quad (16)$$

From (16), it follows that

$$\dot{V}_i = \dot{V}_{i-1} + z_i \left(g_{i,j} x_{i+1} + \bar{f}_{i,j} + d_{i,j} \right) - z_i^2 - \tilde{\theta}_i \dot{\hat{\theta}}_i \quad (17)$$

where $\bar{f}_{i,j} = f_{i,j} - \dot{\alpha}_{i-1} + z_i$, and $\dot{\alpha}_{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} (g_{k,j} x_{k+1} + f_{k,j} + d_{k,j}) + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \dot{\hat{\theta}}_k + \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k)}} y_d^{(k+1)}$.

Similarly, a MTN can be employed to estimate $\bar{f}_{i,j}$, that is to say, for any given constant $\varepsilon_{i,j}$, there exists a MTN $\theta_{i,j}^T P_{m_i}(\mathbf{z}_i)$, such that

$$\bar{f}_{i,j} = \theta_{i,j}^T P_{m_i}(\mathbf{z}_i) + \delta_{i,j}(\mathbf{z}_i), \quad |\delta_{i,j}(\mathbf{z}_i)| \leq \varepsilon_{i,j} \quad (18)$$

where $\mathbf{z}_i = [z_1, \dots, z_i]^T$ is the input vector and $\delta_{i,j}(\mathbf{z}_i)$ is the estimate error.

Combining (17) with (18) and repeating the procedure taken in Step 1, we have

$$\begin{aligned} \dot{V}_i \leq & \sum_{k=1}^{i-1} g_{k,j} z_k z_{k+1} - \sum_{k=1}^{i-1} r_k z_k^2 + \frac{1}{2} \sum_{k=1}^{i-1} (\varepsilon_{k,j}^2 + \bar{d}_{k,j}^2) \\ & + \sum_{k=1}^{i-1} \frac{1}{2} s_k^2 + \sum_{k=1}^{i-1} \tilde{\theta}_k \left(\frac{1}{2s_k^2} z_k^2 P_{m_k}^T P_{m_k} - \dot{\hat{\theta}}_k \right) \\ & + g_{i,j} z_i z_{i+1} + g_{i,j} z_i \alpha_i + \frac{1}{2} \varepsilon_{i,j}^2 + \frac{1}{2} \bar{d}_{i,j}^2 + \frac{1}{2} s_i^2 \\ & + \frac{1}{2s_i^2} z_i^2 \theta_i P_{m_i}^T P_{m_i} - \tilde{\theta}_i \dot{\hat{\theta}}_i \end{aligned} \quad (19)$$

Taking the intermediate control signal α_i as

$$\alpha_i = -\frac{1}{b_m} \left(r_i z_i + \frac{1}{2s_i^2} z_i \hat{\theta}_i P_{m_i}^T P_{m_i} \right) \quad (20)$$

where $r_i > 0$ and $s_i > 0$ are design parameters.

Then, we have

$$g_{i,j} z_i \alpha_i \leq -r_i z_i^2 - \frac{1}{2s_i^2} z_i^2 \hat{\theta}_i P_{m_i}^T P_{m_i} \quad (21)$$

Combining (19) and (21), we have

$$\begin{aligned} \dot{V}_i \leq & \sum_{k=1}^i g_{k,j} z_k z_{k+1} - \sum_{k=1}^i r_k z_k^2 + \frac{1}{2} \sum_{k=1}^i (\varepsilon_{k,j}^2 + \bar{d}_{k,j}^2) \\ & + \sum_{k=1}^i \frac{1}{2} s_k^2 + \sum_{k=1}^i \tilde{\theta}_k \left(\frac{1}{2s_k^2} z_k^2 P_{m_k}^T P_{m_k} - \dot{\hat{\theta}}_k \right) \end{aligned} \quad (22)$$

Step n : Define the new variable $z_n = x_n - \alpha_{n-1}$.

Consider the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{\theta}_n^2 \quad (23)$$

From (23), it follows that

$$\dot{V}_n = \dot{V}_{n-1} + z_n \left(\phi(u) + \bar{f}_{n,j} + d_{n,j} \right) - z_n^2 - \tilde{\theta}_n \dot{\hat{\theta}}_n \quad (24)$$

where $\bar{f}_{n,j} = f_{n,j} - \dot{\alpha}_{n-1} + z_n$, and $\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} (g_{k,j} x_{k+1} + f_{k,j} + d_{k,j}) + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \dot{\hat{\theta}}_k + \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(k)}} y_d^{(k+1)}$.

Similarly, for any given constant $\varepsilon_{n,j} > 0$, there exists a MTN $\theta_n^T P_{m_n}(\mathbf{z}_n)$, such that

$$\bar{f}_{n,j} = \theta_n^T P_{m_n}(\mathbf{z}_n) + \delta_{n,j}(\mathbf{z}_n), \quad |\delta_{n,j}(\mathbf{z}_n)| \leq \varepsilon_{n,j} \quad (25)$$

where $\mathbf{z}_n = [z_1, \dots, z_n]^T$ is input vector and $\delta_{n,j}(\mathbf{z}_n)$ is the estimate error.

Combining (24) with (25) and repeating the procedure taken in Step i , we give

$$\begin{aligned}
\dot{V}_n &\leq \sum_{k=1}^{n-1} g_{k,j} z_k z_{k+1} - \sum_{k=1}^{n-1} r_k z_k^2 + \frac{1}{2} \sum_{k=1}^{n-1} (\varepsilon_{k,j}^2 + \bar{d}_{k,j}^2) \\
&+ \frac{1}{2} \sum_{k=1}^{n-1} s_k^2 + \sum_{k=1}^{n-1} \tilde{\theta}_k \left(\frac{1}{2s_k^2} z_k^T P_{m_k} P_{m_k} - \hat{\theta}_k \right) \\
&+ z_n \phi(u) + \frac{1}{2} \varepsilon_{n,j}^2 + \frac{1}{2} \bar{d}_{n,j}^2 + \frac{1}{2} s_n^2 + \frac{1}{2s_n^2} z_n^T \theta_n P_{m_n}^T P_{m_n} - \tilde{\theta}_n \hat{\theta}_n
\end{aligned} \quad (26)$$

Taking the intermediate control signal u as

$$u = -\frac{1}{\lambda_m} \left(r_n z_n + \frac{1}{2s_n^2} z_n \hat{\theta}_n P_{m_n}^T P_{m_n} \right) \quad (27)$$

where $r_n > 0$ and $s_n > 0$ are design parameters.

Combining (4) and (27), we have

$$z_n \phi(u) \leq -r_n z_n^2 - \frac{1}{2s_n^2} z_n^T \hat{\theta}_n P_{m_n}^T P_{m_n} \quad (28)$$

Combining (26), (27) and (28), it follows that

$$\begin{aligned}
\dot{V}_n &\leq \sum_{k=1}^{n-1} g_{k,j} z_k z_{k+1} + \frac{1}{2} \sum_{k=1}^n (\varepsilon_{k,j}^2 + \bar{d}_{k,j}^2 + s_k^2) \\
&- \sum_{k=1}^n r_k z_k^2 + \sum_{k=1}^n \tilde{\theta}_k \left(\frac{1}{2s_k^2} z_k^T P_{m_k} P_{m_k} - \hat{\theta}_k \right)
\end{aligned} \quad (29)$$

In summary, the design procedure of the MTN-based controller is shown in Figure 1.

Remark 3.2: According to (17) and (18), it can be seen that all unknown functions of each step of backstepping are lump into a unknown function, and only one MTN is employed to compensate this unknown function. Therefore, the computational burden is significantly reduced.

Stability analysis

Theorem 3.1: Under Assumptions 2.1–2.3, consider the switched nonlinear system (1) with sector input defined as (2) and (3). If the control input u designed as (27), the intermediate virtual control signals $\alpha_i (i = 1, \dots, n-1)$ chosen as (20), and the parameter adaptive laws $\hat{\theta}_i (i = 1, \dots, n)$ described as

$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{1}{2s_i^2} z_i^T P_{m_i}^T P_{m_i} \quad (30)$$

where r_i , η_i and s_i are positive design parameters. Then, the proposed adaptive tracking control scheme can guarantee that all signals in the closed-loop system can be guaranteed to be bounded and tracking error converges to a small domain around the origin.

Proof: According to the design process of the controller, consider the following Lyapunov function

$$V = V_n = \frac{1}{2} \sum_{k=1}^n z_k^2 + \frac{1}{2} \sum_{k=1}^n \tilde{\theta}_k^2 \quad (31)$$

From (29), it follows that

$$\begin{aligned}
\dot{V}_n &\leq \sum_{k=1}^{n-1} g_{k,j} z_k z_{k+1} + \frac{1}{2} \sum_{k=1}^n (\varepsilon_{k,j}^2 + \bar{d}_{k,j}^2 + s_k^2) - \sum_{k=1}^n r_k z_k^2 \\
&+ \sum_{k=1}^n \tilde{\theta}_k \left(\frac{1}{2s_k^2} z_k^T P_{m_k} P_{m_k} - \hat{\theta}_k \right)
\end{aligned} \quad (32)$$

Substituting (30) into (32) gives

$$\dot{V} \leq -\sum_{k=1}^n c_k z_k^2 + \frac{1}{2} \sum_{k=1}^n (\varepsilon_{k, \max}^2 + \bar{d}_{k, \max}^2 + s_k^2) + \sum_{k=1}^n \eta_k \tilde{\theta}_k \hat{\theta}_k \quad (33)$$

where $c_k = r_k - b_M > 0$, $\varepsilon_{k, \max} = \max\{\varepsilon_{k,j} | j \in M\}$ and $\bar{d}_{k, \max} = \max\{\bar{d}_{k,j} | j \in M\}$.

On the other hand, the following inequality can be obtained

$$\sum_{k=1}^n \eta_k \tilde{\theta}_k \hat{\theta}_k \leq -\frac{\bar{\eta}_k}{2} \sum_{k=1}^n \tilde{\theta}_k^2 + \frac{1}{2} \sum_{k=1}^n \eta_k \theta_k^2 \quad (34)$$

where $\bar{\eta}_k = \min\{\eta_k | k = 1, \dots, n\}$.

Substituting (34) into (33) gives

$$\begin{aligned}
\dot{V} &\leq -\sum_{k=1}^n c_k z_k^2 + \frac{1}{2} \sum_{k=1}^n (\varepsilon_{k, \max}^2 + \bar{d}_{k, \max}^2 + s_k^2) \\
&- \frac{\bar{\eta}_k}{2} \sum_{k=1}^n \tilde{\theta}_k^2 + \frac{1}{2} \sum_{k=1}^n \eta_k \theta_k^2
\end{aligned} \quad (35)$$

Let $a_0 = \min\{2c_k, \bar{\eta}_k | k = 1, \dots, n\}$ and

$b_0 = \frac{1}{2} \sum_{k=1}^n (\varepsilon_{k, \max}^2 + \bar{d}_{k, \max}^2 + s_k^2) + \frac{1}{2} \sum_{k=1}^n \eta_k \theta_k^2$, yields

$$\dot{V} \leq -a_0 V + b_0 \quad (36)$$

Solving inequality (36), we can get the following inequality

$$0 \leq V(t) \leq V(0)e^{-a_0 t} + \frac{b_0}{a_0} \quad (37)$$

The above inequality means that $V(t)$ is eventually bounded by $\frac{b_0}{a_0}$. Thus, one can obtain that all signals in the closed-loop system can be guaranteed to be bounded and the tracking error converges to a small domain around the origin.

Remark 3.2: It is worth pointing out that the satisfactory closed-loop stability with suitable tracking control performance can be achieved by properly adjusting design parameters r_i , η_i and s_i . Speaking specifically, by increasing the value of r_i , η_i and decreasing the value of $\varepsilon_{i,j}$ and s_k , the tracking error could be decreased. Meanwhile, doing so would also increase the control signal. Therefore, the parameters should be selected appropriately in practical application.

Simulation examples

In this section, two numerical examples are given to demonstrate the effectiveness of the proposed approach.

Example 1: Consider the following switch stochastic nonlinear system

$$\begin{cases} \dot{x}_1 = g_{1,\sigma(t)}(\bar{x}_1)x_2 + f_{1,\sigma(t)}(\bar{x}_1) + d_{1,\sigma(t)} \\ \dot{x}_2 = g_{2,\sigma(t)}(\bar{x}_2)x_3 + f_{2,\sigma(t)}(\bar{x}_2) + d_{2,\sigma(t)} \\ \dot{x}_3 = \varphi(u) + f_{3,\sigma(t)}(\bar{x}_3) + d_{3,\sigma(t)} \\ y = x_1 \end{cases} \quad (38)$$

where $\varphi(u) = (2 + 0.5 \sin(u))u$, $\sigma(t): [0, \infty) \rightarrow M = \{1, 2\}$, the unknown nonlinear functions are $g_{1,1} = 1$, $g_{1,2} = 1 + \frac{x_1^2}{x_1^2 + 1}$, $f_{1,1} = -2x_1 e^{-0.5x_1}$, $f_{1,2} = -2 \sin(x_1) e^{-0.5x_1}$, $g_{2,1} = 1$, $g_{2,2} = 1.5 + 0.5 \sin(x_1 x_2)$, $f_{2,1} = x_1 \cos(x_2^2)$, $f_{2,2} = x_1 \sin(x_2^2)$, $f_{3,1} = x_2 x_3$, $f_{3,2} = x_1 x_2 x_3$; the bounded disturbance are $d_{1,1} = 0.2 \sin t \cos t$, $d_{2,1} = 0.2 \sin t$, $d_{3,1} = 0.2 \cos t$, $d_{1,2} = 0.1 \sin t \cos t$, $d_{2,2} = 0.1 \cos t$, $d_{3,2} = 0.1 \sin t$. The reference signal $y_d = 0.5(\sin t + \sin(0.5t))$.

According to Theorem 3.1, the virtual control signals $\alpha_i (i = 1, 2)$ and the true control law u are designed as

$$\alpha_i = -\frac{1}{b_m} \left(r_i z_i + \frac{1}{2s_i^2} z_i \hat{\theta}_i^T P_{m_i}^T P_{m_i} \right), \quad i = 1, 2$$

$$u = -\frac{1}{\lambda} \left(r_3 z_3 + \frac{1}{2s_3^2} z_3 \hat{\theta}_3^T P_{m_3}^T P_{m_3} \right)$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$ and $z_1 = [z_1]^T$, $z_2 = [z_1, z_2]^T$, $z_3 = [z_1, z_2, z_3]^T$. The adaptive laws are designed as

$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{1}{2s_i^2} z_i^T P_{m_i}^T P_{m_i}, \quad i = 1, 2, 3$$

From system (38), we can get $b_m = 1$ and $b_M = 2$, $\bar{\lambda} = 2.5$. In simulation, the design parameters are chosen as: $r_1 = 15$, $r_2 = 5$, $r_3 = 20$, $\eta_1 = 4$, $\eta_2 = 2$, $\eta_3 = 0.1$, $s_1 = 1$, $s_2 = 1$ and $s_3 = 1$. The simulation results are displayed in Figures 2–7.

Figure 2 shows the system output y (dotted line) and the reference signal y_d (solid line), from which we can see that a good tracking performance has been achieved. Figure 3 displays the trajectories of u and $\varphi(u)$. Figure 4 depicts that the tracking error converges to a small domain around the origin. Figure 5 and Figure 7 shows that the signals in the closed-loop system, such as state variables x_2 , x_3 and adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, are bounded. Figure 6 displays the switching signal $\sigma(t)$.

Example 2: According to Niu et al. (2019), a class of closed, continuously stirred tank, chemical reactor with two modes of feed stream has the following form

$$\begin{cases} \dot{x}_1 = g_{1,\sigma(t)}(\bar{x}_1)x_2 + f_{1,\sigma(t)}(\bar{x}_1) + d_{1,\sigma(t)} \\ \dot{x}_2 = \varphi(u) + f_{2,\sigma(t)}(\bar{x}_2) + d_{2,\sigma(t)} \\ y = x_1 \end{cases} \quad (39)$$

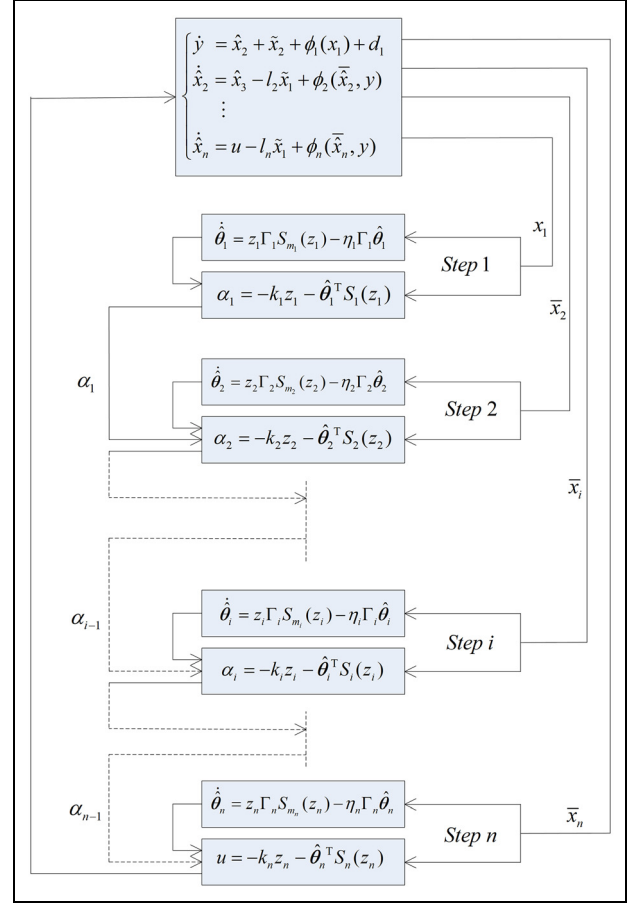


Figure 1. Block diagram of control system.

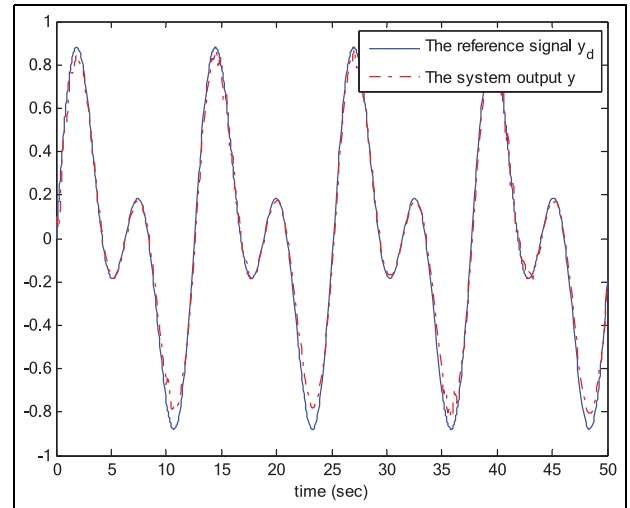


Figure 2. System output y and reference signal y_d of example 1.

where $\varphi(u) = (2 + 0.5 \sin(u))u$, $\sigma(t): [0, \infty) \rightarrow M = \{1, 2\}$, and the unknown nonlinear functions are $g_{1,1}(\bar{x}_1) = g_{1,2}(\bar{x}_1) = 1$, $f_{1,1}(\bar{x}_1) = \frac{1}{2}x_1$, $f_{1,2}(\bar{x}_1) = 2x_1$, $f_{2,1}(\bar{x}_2) =$

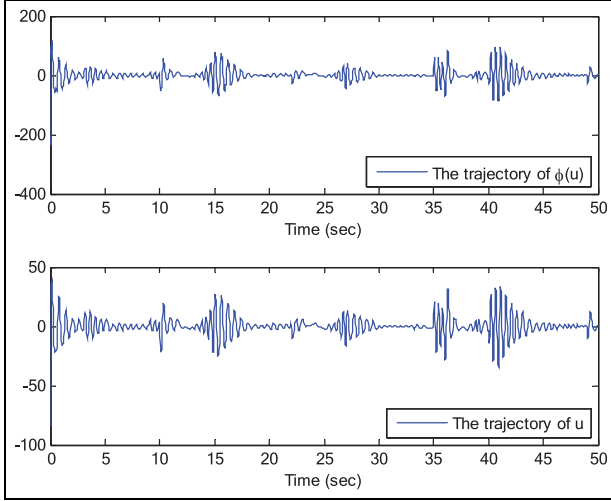


Figure 3. The trajectories of $\phi(u)$ and u of example 1.

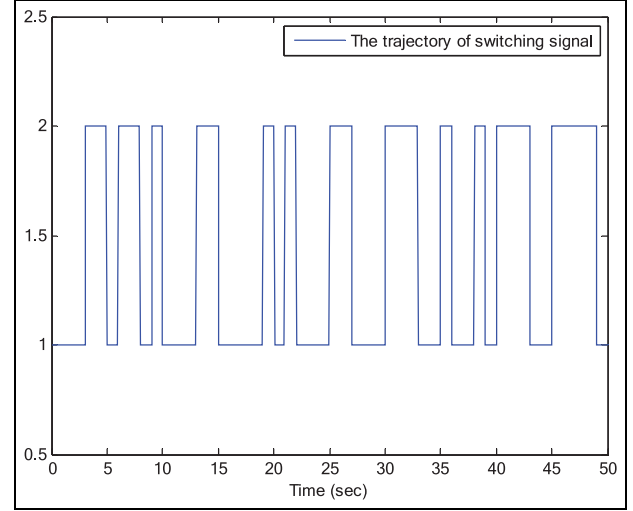


Figure 6. The trajectory of switching signal $\sigma(t)$ of example 1.

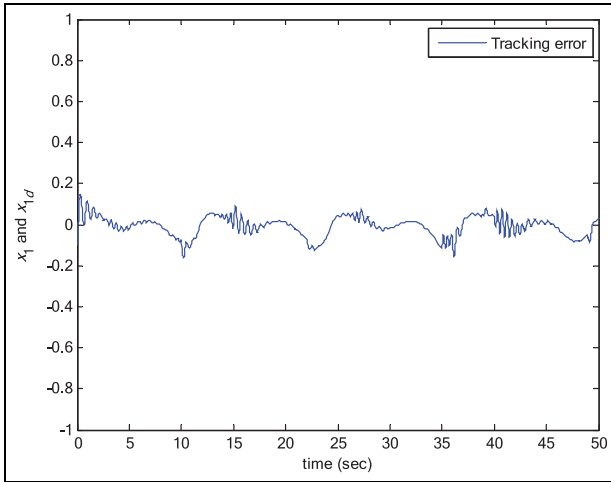


Figure 4. The tracking error of example 1.

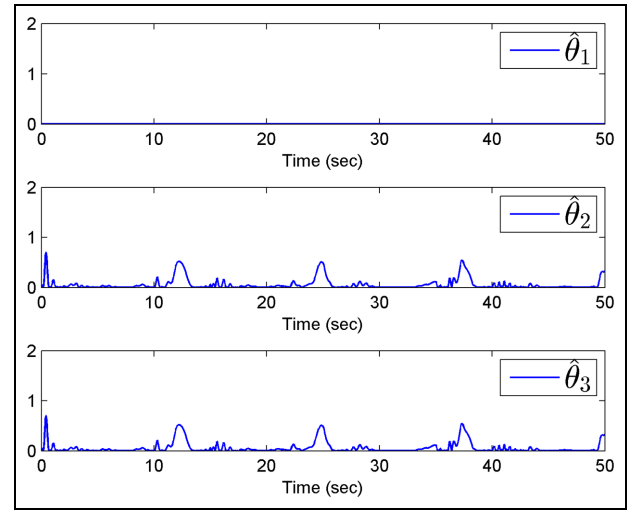


Figure 7. The adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$ of example 1.

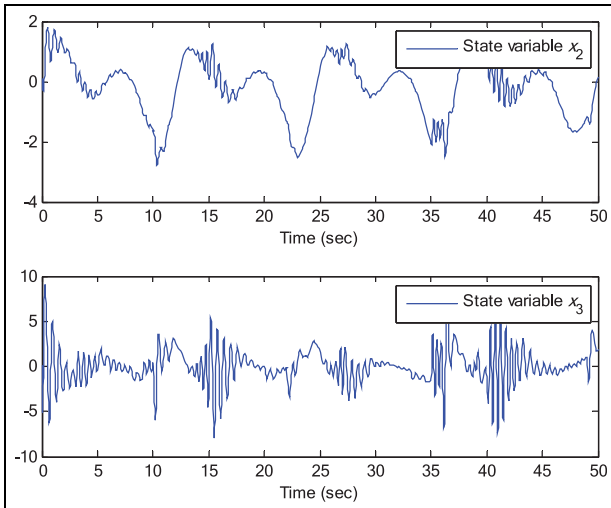


Figure 5. State variables x_2 and x_3 of example 1.

$f_{2,2}(\bar{x}_2) = 0$, $d_{1,1} = 0.1 \sin t$, $d_{1,2} = 0.1 \cos t$ and $d_{2,1} = d_{2,2} = 0$. The reference signal $y_d = 0.5(\sin t + \sin(0.5t))$.

According to Theorem 3.1, the virtual control signal α_1 and the true control law u are designed as

$$\alpha_1 = -\frac{1}{b_m} \left(r_1 z_1 + \frac{1}{2s_1^2} z_1 \hat{\theta}_1^T P_{m_1} P_{m_1} \right)$$

$$u = -\frac{1}{\lambda} \left(r_2 z_2 + \frac{1}{2s_2^2} z_2 \hat{\theta}_2^T P_{m_2} P_{m_2} \right)$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$ and $\mathbf{z}_1 = [z_1]^T$, $\mathbf{z}_2 = [z_1, z_2]^T$. The adaptive laws are designed as

$$\dot{\hat{\theta}}_i = -\eta_i \hat{\theta}_i + \frac{1}{2s_i^2} z_i^2 P_{m_i}^T P_{m_i}, \quad i = 1, 2$$

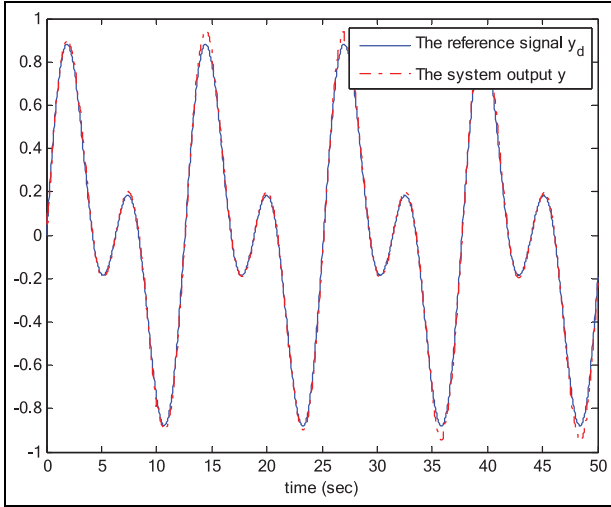


Figure 8. System output y and reference signal y_d of example 2.

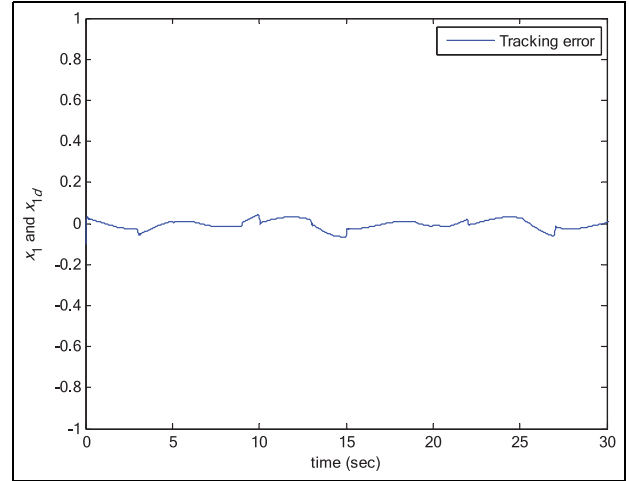


Figure 10. The tracking error of example 2.

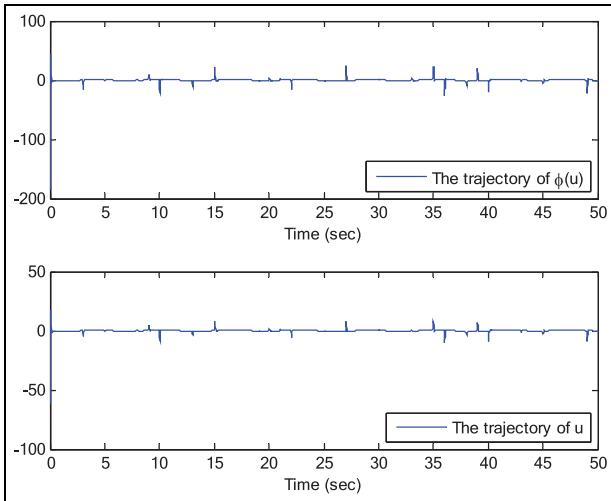


Figure 9. The trajectories of $\phi(u)$ and u of example 2.

From system (38), we can get $b_m = b_M = 1$ and $\bar{\lambda} = 2.5$. In simulation, the design parameters are chosen as: $r_1 = 30$, $r_2 = 20$, $\eta_1 = 4$, $\eta_2 = 0.1$, $s_1 = 1$ and $s_2 = 1$. The simulation results are displayed in Figures 8–12. From Figures 8–12 we can see that the control performance is satisfactory, which further indicate the effectiveness of the method proposed.

Remark 4.1: It is of interest to note that the choice of the design parameters plays an important role in the tracking performance. Although the tracking performances could be reached when the design parameters satisfy $r_i > 0$, $\eta_i > 0$ and $s_i > 0$ according to Theorem 3.1, the design parameters should be selected appropriately to obtain a specific control objective in practice.

Remark 4.2: From the above simulation results, it is clear that: (1) even though the nonlinear systems contain the unknown nonlinearities and control directions, and control

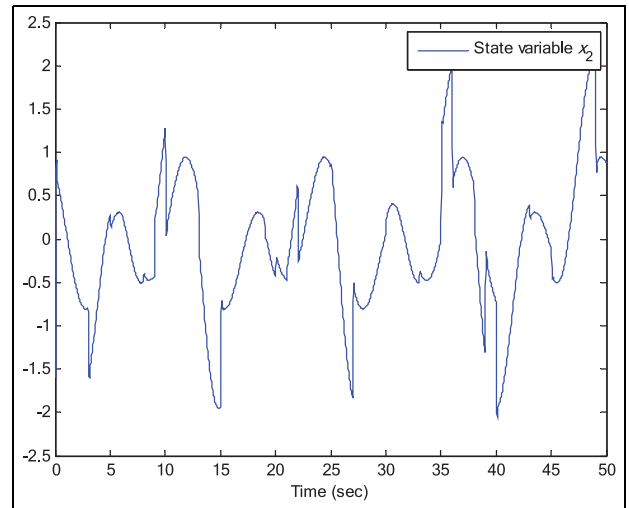


Figure 11. State variable x_2 of example 2.

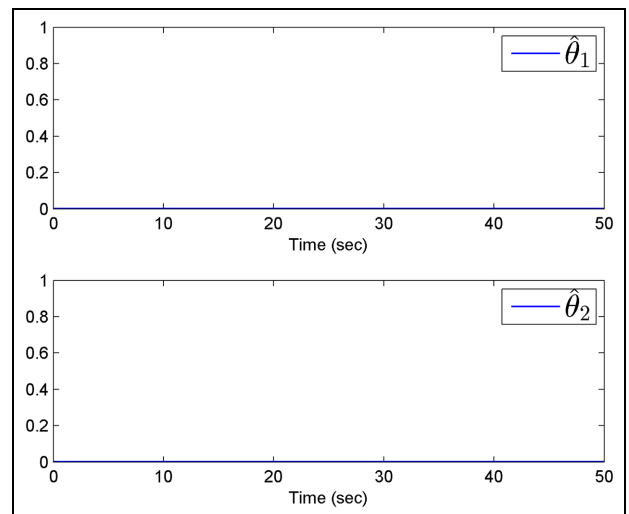


Figure 12. The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ of example 2.

input nonlinearity, the proposed adaptive MTN-based controllers are still to obtain a satisfactory tracking performance; (2) the control method designed in this paper is available to both numerical and practical examples.

Conclusions

In this paper, a new MTN-based adaptive tracking control scheme is proposed for a class of switched nonlinear systems with input nonlinearity. The design procedure is constructive and MTN-backstepping-based. Specifically, the MTNs are used to approximate the unknown nonlinearities, and an MTN-based adaptive tracking controller is developed via backstepping. The proposed controller has the advantages of simple structure and good real time feature. On this foundation, realizes tracking control for switched nonlinear systems with input nonlinearity.


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