Adaptive Multi-Switching-Based Global Tracking Control for Switched Nonlinear Systems With Prescribed Performance

Wen-Jing He, Shan-Liang Zhu, Li-Ting Lu, Wei Zhao, and Yu-Qun Han

***Abstract*— This paper is concerned with the tracking control for a class of switched nonlinear systems subject to prescribed performance. Firstly, a barrier function and a normalized function are introduced to achieve prescribed performance control, which ensures that the tracking error evolves within prescribed boundary. Then, multi-dimensional Taylor network provides a new approach for how to estimate the nonlinearity arising from the backstepping control process. Significantly, the proposed multi-switching-based adaptive controller realizes that all signals in the closed-loop system are globally uniformly ultimately bounded while ensuring asymptotic tracking. Different from most existing network-approximation-based control strategies, the developed method in this paper is not only independent of the initial state, but also can achieve global stability. Finally, it is easy to verify the effectiveness of the proposed control method through two simulations.**

***Note to Practitioners*—This research is motivated by the fact that the control ideas of many practical engineering systems can be provided by making a profound study on switched nonlinear systems. However, most of the existing results focused on semi-global stability of systems. Therefore, this study devotes to develop a novel adaptive prescribed performance control strategy, which can not only ensure global stability of closed-loop system but also realize the asymptotic tracking of the switched nonlinear systems. It is worth noting that the proposed control approach has great significance for many practical systems, such as circuit systems and single-link inverted pendulum systems.**

***Index Terms*— Global control, switched nonlinear systems, prescribed performance, multi-dimensional Taylor network.**

I. Introduction

IT IS widely known that a single system model cannot be used to describe the system with interrelated controlled objects. Therefore, the control research of switched systems consisting of several continuous subsystems and discrete switching signals has been a hot topic of discussion [1], [2], [3]. There is an acknowledged fact that the control ideas of many practical systems can be provided by making a profound study on switched systems, such as circuit systems [4], aero-engine systems [5] and automobile test-driving systems [6]. In order to cope with the control issue of switched nonlinear systems, many fruitful control methods have been developed, such as H∞ control [7], sliding mode control [8], sampled-data control [9], robust control [10] and adaptive backstepping control [11]. Among them, adaptive backstepping control has been given close attention by the control community because of its wide applications. However, if there are strong nonlinearity and uncertainty in the controlled system, it is difficult to achieve the control object by only relying on the aforementioned control approaches.

The main work of neural networks (NNs) and fuzzy logic systems (FLSs) is to estimate complex nonlinearity, which makes the approximation-based adaptive controller powerful enough to compensate for the control requirements of complex nonlinear systems. Therefore, fruitful NN-based and FLS-based achievements have been proposed to solve the tracking control issues of various nonlinear systems, such as uncertain nonlinear systems [12], [13], large-scale nonlinear systems [14], [15], switched nonlinear systems [16], [17], [18] and stochastic nonlinear systems [19], [20]. Significantly, a new way is given by the multi-dimensional Taylor network (MTN) for estimating nonlinearity, which arises in the backstepping process [21], [22], [23], [24]. It is exciting that the application of MTN in quite a few switched nonlinear systems is a worthwhile task, such as switched time-delay nonlinear systems [25], switched stochastic nonlinear systems [26], switched hysteresis nonlinear systems [27] and switched large-scale nonlinear systems [28]. Despite the fact that the research on MTN-based control of switched nonlinear systems is relatively complete, how to achieve the prescribed performance of the controlled system is widely recognized as a challenging but fascinating topic.

In many practical applications, it is inevitable that the system output is required to track the desired signal in a predetermined way, such as missile interception, self-driving cars, and aircraft docking. Therefore, prescribed performance control has been given a much higher attention. Not only do the typical prescribed performance control methods achieve steady-state performance, but they also meet predefined transients, such as funnel control [29], [30], [31], [32] and prescribed performance bound (PPB) control [33], [34], [35], [36]. However,

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it should be pointed out that these control methods depend on the initial conditions. For example, a key limitation of the prescribed performance control strategy based on network approximation mentioned in [36] is that the performance function is required to meet the conditions $-\delta_{\min}\mu(0) < y(0) - y_d(0) < \delta_{\max}\mu(0)$, with $\delta_{\min}$ and $\delta_{\max}$ are positive parameters. Obviously, the system output $y(0)$ and the desired signal $y_d(0)$ have a crucial impact on the selection of the prescribed performance function $\mu(0)$. In general, the conditions imposed are so restrictive that the application of prescribed performance control has become a significant obstacle. Significantly, with the aim of ensuring that the proposed prescribed performance controller bears no relationship to the initial conditions, [37] considered a tuning function, which modified the predetermined tracking trajectory. In addition, [38] designed a prescribed function independent of the desired signal and the initial conditions, and introduced a barrier function and a normalized function to convert the constrained systems into the unconstrained systems. Sad to say, the aforementioned studies cannot achieve network approximation and globally stable control at the same time. Consequently, how to balance estimation technique and global control is facing a big challenge. Based on the above discussion, it is an urgent problem to develop a novel MTN-based prescribed performance control strategy to achieve global asymptotic tracking of switched nonlinear systems.

In consideration of the difficulty in acquiring global control by MTN technique and the necessity of getting rid of the restrictions on the initial state, this paper is devoted to exploring the MTN-based global tracking control method for the switched nonlinear systems with prescribed performance. The contribution of this paper is highlighted by the following three aspects:

1) Unlike the global control method under prescribed performance reported in [39] and [40], this paper is the first extraordinary work to investigate the global asymptotic tracking control of switched nonlinear systems using MTN estimation technique, which has challenged the conventional understanding of network-approximation-based global control. In the design process of backstepping control, an adaptive multi-switching controller is constructed to deal with the coupling problem of estimation technique and global control.
2) Different from the prescribed functions considered in [34] and [41], this paper designs a special prescribed function by introducing a time-varying scaling function, which is independent of the given desired signal and the initial conditions of the systems. For the sake of serving the purpose of transforming the constrained systems into the unconstrained systems, a barrier function and a normalized function are designed. The prescribed performance control strategy proposed in this paper makes sure that the tracking error not only develops within the prescribed range but also asymptotically converges to zero.
3) For the purpose of resulting in optimal control performance, on the basis of considering the traditional adaptive control law of dominant approximation, an extra adaptive law is introduced, and a continuous and differentiable switching function is constructed to design the desired controller.

II. Problem Formulation and Preliminaries

A. System Description

Considering the switched nonlinear system with unknown control directions, which is described as

$$\begin{cases} \dot{x}_i = g_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i)\, x_{i+1} + f_{i,\sigma(t)}(\bar{\boldsymbol{x}}_i) \\ \dot{x}_n = g_{n,\sigma(t)}(\bar{\boldsymbol{x}}_n)\, u + f_{n,\sigma(t)}(\bar{\boldsymbol{x}}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, n-1$, and $\sigma(t) : R_+ \rightarrow M = \{1, 2, \cdots, m\}$ defines switching signal, and m is denoted as the number of subsystems. $\bar{\boldsymbol{x}}_n = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in R^n$ is defined as the state vector of the system with $\bar{\boldsymbol{x}}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i$. $u, y \in R$ denote the control input and the output of the system, respectively. For $i = 1, 2, \cdots, n$ and $k \in M$, $g_{i,k}(\cdot)$ represents the control coefficient, and its symbol indicates the control direction of the i-th channel, $f_{i,k}(\cdot)$ expresses the smooth and unknown function.

Remark 1: For the past few years, the issue of prescribed performance control has been extensively used in the single model system. However, switched nonlinear systems can better describe the characteristics of quite a few physical models and practical systems, such as bipedal robots [42] and automotive systems [43]. On the other hand, the existing approximation-based works usually result in errors, which can significantly degrade system performance. Therefore, motivated by the above problems, this paper develops the MTN-based global control strategy for switched nonlinear systems with prescribed performance.

For the system (1), the control objectives of this paper are described as the following three points:

(i) All of the signals in the closed-loop system are guaranteed to be globally ultimately uniformly bounded (GUUB);

(ii) The given desired signal y_d can be tracked asymptotically by the system output y.

(iii) The tracking error $e = y - y_d$ can be independent of the initial conditions and can evolve within prescribed boundary $\wp := \{(t, e) \in R_+ \times R \,|\, E(-\vartheta(t)) < e < E(\vartheta(t))\}$, with $E(\vartheta(t))$ is the prescribed function, $\vartheta(t)$ defines the time-varying function, and their expression will be given below.

Based on the above control objectives, in order to design an excellent controller, some Assumptions and Lemmas are introduced.

Assumption 1: The desired signal y_d and its derivatives up to the n-th order are bounded and continuous.

Assumption 2: For the continuous and unknown function $f_i(\cdot)$, there exist a known positive function $h_i(\cdot)$ and an unknown constant λ_i that satisfy $|f_i(\cdot)| \leq \lambda_i h_i(\cdot)$.

Assumption 3: For the function $g_{i,k}(\cdot)$ in the system (1), there exist constants g_m and g_M such that $0 < g_m \leq |g_{i,k}(\cdot)| \leq g_M < \infty$ can be ensured. In general, the sign of $g_{i,k}(\cdot)$ is assumed as positive.

Remark 2: Among the existing research achievements [11], [12], [44], [45], Assumption 1 and Assumption 3 are general assumptions for the control problems of nonlinear systems

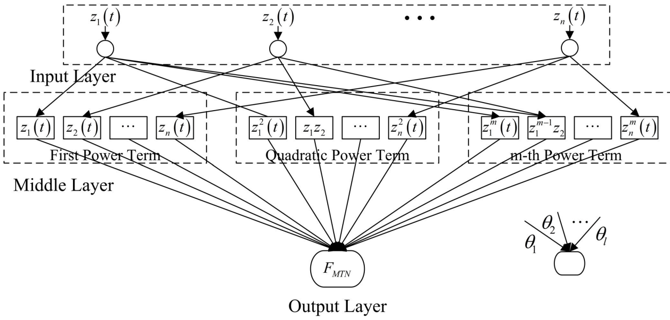

Fig. 1. The network structure of MTN.

with unknown control directions. In addition, the goal of Assumption 2 is to estimate the upper bounds for some unknown functions.

Lemma 1 [38]: For any constant $c > 0$ and any scalar variable $x \in (-1, 1)$, the function $E(x) = \frac{\sqrt{c}x}{\sqrt{1-x^2}}$ is strictly monotonically increasing.

Lemma 2 [27]: For any $\tau > 0$, the continuous nonlinear function $F(Z)$ defined on a compact set Ω_Z can be estimated by one MTN, which can be expressed as

$$F(Z) = \boldsymbol{\theta}^{*\mathrm{T}} P_{m_n}(Z) + \varepsilon(Z) \tag{2}$$

where $\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}\in R^l} \left\{ \sup_{F(Z)\in\Omega_Z} \left| F(Z) - \boldsymbol{\theta}^{\mathrm{T}} P_{m_n}(Z) \right| \right\}$, and $\boldsymbol{\theta}$ is defined as the weight vector of MTN. $P_{m_n}(Z) = \left[z_1, z_2, \cdots, z_n, z_1^2, z_1 z_2, \cdots, z_n^2, \cdots, z_1^m, \cdots, z_n^m\right]^{\mathrm{T}} \in R^l$ represents the middle layer vector of MTN. $Z = [z_1, z_2, \cdots, z_n]^{\mathrm{T}} \in R^n$ indicates the input layer vector of MTN. $\varepsilon(Z)$ denotes the estimation error of MTN and $|\varepsilon(Z)| < \tau$.

Remark 3: MTN is a network structure as shown in Fig. 1, which is composed of input layer, output layer and middle layer. The middle layer is the high power combination of input layer vector. MTN demonstrates to estimate nonlinear functions with arbitrary accuracy, which can be drawn from [21].

Definition 1: Considering the nonlinear system with the following form

$$\dot{x} = f(x, u, t) \tag{3}$$

where x denotes the system state, u defines the system input. If for any $t_0 \geq 0$ and any constant $c \in (0, b)$, there exist positive constants a, b and $T = T(a, b)$ such that $|x(t_0)| \leq c \Rightarrow |x(t)| \leq a, \forall t \geq t_0 + T$ holds, then the solution of the system (3) is called GUUB.

Lemma 3: For the nonlinear system (3), supposing that $D = R^n$ is the domain of definition and $V : [0, \infty) \times D \to R$ is a continuous differentiable positive definite function. For the given $\varepsilon > 0$, selecting $r \in (0, \varepsilon]$ such that $B_r = \{x \in R^n \,|\, \|x\| \leq r\} \subset D$. If there exist class κ functions β_1 and β_2 defined on D, such that the following four points hold:

(i) $\beta_1(\|x\|) \leq V(x) \leq \beta_2(\|x\|)$;

(ii) $V(0) = 0$, $V(x) > 0, \forall x \neq 0$;

(iii) $\dot{V}(x) < 0, \forall x \neq 0$;

(iv) $\|x\| \to \infty \Rightarrow V(x) \to \infty$.

Then, the solution of the system (3) is globally asymptotically stable.

Proof: The proof idea of Lemma 3 comes from [46].

B. Processing of the Tracking Error

In order to achieve the control objective (iii), according to [38], it is instructive to introduce the following important functions.

1) Prescribed Function: For a known and positive constant c, the following prescribed function $E(\vartheta(t))$ is constructed

$$E(\vartheta(t)) = \frac{\sqrt{c}\vartheta(t)}{\sqrt{1 - \vartheta^2(t)}} \tag{4}$$

where $\vartheta(t) = \frac{1}{\gamma(t)}$, $\gamma(t) : [0, \infty) \to R$ denotes a time-varying scaling function, which has the following two properties:

(i) $\gamma(t)$ has continuous derivatives up to order n, and $\gamma^{(l)}, l = 0, 1, \cdots, n$ are piecewise continuous and bounded;

(ii) For $\forall t \geq 0$, $\gamma(t)$ is a monotonically increasing function with $\gamma(0) = 1$, and $\lim_{t\to\infty} \gamma(t) = \frac{1}{r_f}$ with $0 < r_f << 1$ is a constant.

Therefore, it can be concluded that $\vartheta(0) = \frac{1}{\gamma(0)} = 1$, $\lim_{t\to\infty} \vartheta(t) = \lim_{t\to\infty} \frac{1}{\gamma(t)} = r_f$, $E(\vartheta(0)) = E(1) \to \infty$ and $\lim_{t\to\infty} E(\vartheta(t)) = r_f$, with $c = 1 - r_f^2$.

2) Normalized Function: A novel error transformation is considered, which can be expressed as follows

$$\varphi(e(t)) = \frac{e(t)}{\sqrt{e^2(t) + c}} \tag{5}$$

Namely, $\varphi(e(t))$ can be called the normalized function based on tracking error, and the following three points can be guaranteed:

(i) $\varphi(e(t))$ is strictly monotonical;

(ii) For $\forall e \in R$, $\varphi(e(t)) \in (-1, 1)$, and $\lim_{e\to+\infty} \varphi(e(t)) \to 1$, $\lim_{e\to-\infty} \varphi(e(t)) \to -1$;

(iii) If there is a constant $\bar{\varphi}$ such that $|\varphi(e(t))| \leq \bar{\varphi} < 1$ holds, then the boundedness of $e = \frac{\sqrt{c}\varphi}{\sqrt{1-\varphi^2}} = E(\varphi)$ can be ensured.

3) Barrier Function: A barrier function is designed, which is expressed in the following form

$$\xi(t) = \frac{\upsilon(t)}{1 - \upsilon^2(t)} \tag{6}$$

where $\upsilon(t) = \gamma(t)\varphi(e(t))$, $\upsilon(0) = \gamma(0)\varphi(e(0)) = \varphi(e(0)) \in (-1, 1)$.

For tracking error $e(t)$ with arbitrary bounded initial value, if $\xi(t)$ is bounded, then for $\forall t \geq 0$, $|\upsilon(t)| \leq \delta < 1$ holds with $\delta > 0$ is a constant.

4) Discussion: Based on the definition of the above functions, it can be obtained that $-\vartheta = -\frac{1}{\gamma} < -\frac{\delta}{\gamma} \leq \varphi \leq \frac{\delta}{\gamma} < \frac{1}{\gamma} = \vartheta$. Then, with the help of Lemma 1, $E(-\vartheta) < E(\varphi) = e < E(\vartheta)$ can be acquired.

To sum up, the problem that the tracking error is limited to range $\wp$ will be transformed into a bounded problem of $\xi(t)$.

III. Multi-Switching-Based Adaptive MTN Controller Design

First of all, with the aim of designing an adaptive global controller based on multi-switching, the following function is constructed, and its main idea comes from [44]

$$K(Z) := \prod_{i=1}^{l} \kappa(\phi_i) \tag{7}$$

where

$$\kappa = \begin{cases} 1, & |\phi| \le s_1 \\ 0, & |\phi| \ge s_2 \\ \cos^n\left(\frac{\pi}{2}\sin^n\left(\frac{\pi\left(|\phi|^2 - s_1^2\right)}{2\left(s_2^2 - s_1^2\right)}\right)\right), & else, \end{cases}$$
$$Z = [\phi_1, \phi_2, \cdots, \phi_l]^{\mathrm{T}} \in R^l, \quad s_1, s_2$$

are constants and $s_2 > s_1 > 0$, which indicate switching lower bound and switching upper bound of variable ϕ, respectively.

Secondly, the following coordinate transformations are defined

$$\begin{cases} z_1 = \xi \\ z_i = x_i - \alpha_{i-1}, \quad i = 2, \cdots, n \end{cases} \tag{8}$$

where α_{i-1} is the virtual control signal, which value will be given in the backstepping design later.

Thirdly, for the design parameter $*$, the estimation error $\tilde{*} = * - \hat{*}$ is denoted, with $\hat{*}$ represents the estimated value of $*$.

Step 1: The candidate Lyapunov function V_1 with the following form is constructed

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{1,k} + \frac{1}{2}\tilde{\lambda}_{1,k}^2 + \frac{1}{2}\tilde{\tau}_{1,k}^2 \tag{9}$$

With the help of (1), (5), (6), $x_2 = z_2 + \alpha_1$ and $v = \gamma\varphi(e)$, the time derivative of the candidate Lyapunov function V_1 can be described as

$$\dot{V}_1 = \left[p_1 g_{1,k}(z_2 + \alpha_1) + F_{1,k}(Z_1)\right] z_1 - \tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{1,k} - \tilde{\lambda}_{1,k}\dot{\hat{\lambda}}_{1,k} - \tilde{\tau}_{1,k}\dot{\hat{\tau}}_{1,k} \tag{10}$$

where $Z_1 = \left[x_1, y_d, \dot{y}_d\right]^{\mathrm{T}}$, $p = \frac{1+v^2}{(1-v^2)^2}$, $p_1 = \frac{p\gamma c}{(e^2+c)^{\frac{3}{2}}}$, $p_2 = p\dot{\gamma}\varphi$ and $F_{1,k}(Z_1) = p_1\left(f_{1,k} - \dot{y}_d\right) + p_2$.

According to Lemma 2, $F_{1,k}(Z_1)$ as a combination of continuous unknown nonlinear functions can be estimated by MTN. Namely, for given $\tau_{1,k} > 0$, there exists one MTN as $\boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1}(Z_1)$, such that

$$F_{1,k}(Z_1) = \boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1}(Z_1) + \varepsilon_{1,k}(Z_1), \quad \left|\varepsilon_{1,k}\right| < \tau_{1,k} \tag{11}$$

where $\varepsilon_{1,k}(Z_1)$ represents the estimation error.

Based on Assumption 2, there are a known and positive function $h_{1,k}(Z_1)$ and an unknown constant $\lambda_{1,k}$ such that the following inequality holds

$$\left|F_{1,k}(Z_1)\right| \le \lambda_{1,k} h_{1,k}(Z_1) \tag{12}$$

Combining (10), (11) with (12), the time derivative of the candidate Lyapunov function V_1 can be rewritten as

$$\begin{aligned} \dot{V}_1 \le & \left(\boldsymbol{\theta}_{1,k}^{\mathrm{T}} P_{m_1}(Z_1) + \tau_{1,k}\right) K(Z_1) z_1 - \tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{1,k} \\ & - \tilde{\lambda}_{1,k}\dot{\hat{\lambda}}_{1,k} - \tilde{\tau}_{1,k}\dot{\hat{\tau}}_{1,k} + p_1\left(g_{1,k}z_2 + g_{1,k}\alpha_1\right) z_1 \\ & + (1 - K(Z_1))\lambda_{1,k}h_{1,k}(Z_1) z_1 \end{aligned} \tag{13}$$

The virtual control signal α_1 with the following form is designed

$$\alpha_1 = -\frac{1}{p_1 g_m}\left[r_1 z_1 + K(Z_1)\alpha_1^f + (1 - K(Z_1))\alpha_1^r\right] \tag{14}$$

where $r_1 > 0$ is a constant, and $\alpha_1^f = \hat{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}} P_{m_1}(Z_1) + \hat{\tau}_{1,k}$, $\alpha_1^r = \hat{\lambda}_{1,k}h_{1,k}(Z_1)$.

By means of Assumption 3, the virtual control signal α_1 and (13), the time derivative of the candidate Lyapunov function V_1 can be described as

$$\begin{aligned} \dot{V}_1 \le & p_1 g_{1,k} z_1 z_2 - r_1 z_1^2 + (1 - K(Z_1)) z_1 \tilde{\lambda}_{1,k} h_{1,k} \\ & + K(Z_1) z_1 \tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}} P_{m_1} + K(Z_1) z_1 \tilde{\tau}_{1,k} - \tilde{\boldsymbol{\theta}}_{1,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{1,k} \\ & - \tilde{\lambda}_{1,k}\dot{\hat{\lambda}}_{1,k} - \tilde{\tau}_{1,k}\dot{\hat{\tau}}_{1,k} \end{aligned} \tag{15}$$

Based on the above analysis, three adaptive laws are designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_{1,k} = K(Z_1) z_1 P_{m_1} \\ \dot{\hat{\lambda}}_{1,k} = (1 - K(Z_1)) z_1 h_{1,k} \\ \dot{\hat{\tau}}_{1,k} = K(Z_1) z_1 \end{cases} \tag{16}$$

By substituting (16) into (15), resulting in the following inequality

$$\dot{V}_1 \le -r_1 z_1^2 + p_1 g_{1,k} z_1 z_2 \tag{17}$$

Step i ($i = 2, 3, \cdots, n-1$): The candidate Lyapunov function V_i is designed as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,k} + \frac{1}{2}\tilde{\lambda}_{i,k}^2 + \frac{1}{2}\tilde{\tau}_{i,k}^2 \tag{18}$$

Combining (1) with $x_{i+1} = z_{i+1} + \alpha_i$, the time derivative of the candidate Lyapunov function V_i can be indicated as

$$\dot{V}_i = \left[g_{i,k}(z_{i+1} + \alpha_i) + F_{i,k}(Z_i)\right] z_i - g_{i-1,k} z_{i-1} z_i - \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{i,k} - \tilde{\lambda}_{i,k}\dot{\hat{\lambda}}_{i,k} - \tilde{\tau}_{i,k}\dot{\hat{\tau}}_{i,k} + \dot{V}_{i-1} \tag{19}$$

where $Z_i = \left[\bar{x}_i, \bar{y}_d^{(i)}, \bar{\hat{\boldsymbol{\theta}}}_{i-1,k}, \bar{\hat{\lambda}}_{i-1,k}, \bar{\hat{\tau}}_{i-1,k}\right]^{\mathrm{T}}$, $F_{i,k}(Z_i) = f_{i,k} - \dot{\alpha}_{i-1} + g_{i-1,k}z_{i-1}$, $\bar{y}_d^{(i)} = \left[y_d, \dot{y}_d, \cdots, y_d^{(i)}\right]^{\mathrm{T}}$, $\bar{\hat{\boldsymbol{\theta}}}_{i-1,k} = \left[\hat{\boldsymbol{\theta}}_{1,k}, \cdots, \hat{\boldsymbol{\theta}}_{i-1,k}\right]^{\mathrm{T}}$, $\bar{\hat{\lambda}}_{i-1,k} = \left[\hat{\lambda}_{1,k}, \cdots, \hat{\lambda}_{i-1,k}\right]^{\mathrm{T}}$, $\bar{\hat{\tau}}_{i-1,k} = \left[\hat{\tau}_{1,k}, \cdots, \hat{\tau}_{i-1,k}\right]^{\mathrm{T}}$.

On the basis of Lemma 2, $F_{i,k}(Z_i)$ as the combination of continuous and unknown nonlinear functions can be estimated by MTN. Namely, for given $\tau_{i,k} > 0$, there is one MTN as $\boldsymbol{\theta}_{i,k}^{\mathrm{T}} P_{m_i}(Z_i)$, such that

$$F_{i,k}(Z_i) = \boldsymbol{\theta}_{i,k}^{\mathrm{T}} P_{m_i}(Z_i) + \varepsilon_{i,k}(Z_i), \quad \left|\varepsilon_{i,k}\right| < \tau_{i,k} \tag{20}$$

where $\varepsilon_{i,k}(Z_i)$ denotes the estimation error.

By fully considering Assumption 2, there exist the unknown constant $\lambda_{i,k}$ and the known and positive function $h_{i,k}(Z_i)$ such that the following inequality holds

$$\left|F_{i,k}(Z_i)\right| \le \lambda_{i,k} h_{i,k}(Z_i) \tag{21}$$

Combining (19), (20) with (21), the time derivative of the candidate Lyapunov function V_i can be rewritten in the following form

$$\begin{aligned}\dot{V}_i \le\ & \dot{V}_{i-1} + K(Z_i)\left(\boldsymbol{\theta}_{i,k}^{\mathrm{T}} P_{m_i}(Z_i) + \tau_{i,k}\right) z_i \\ & + (1 - K(Z_i))\lambda_{i,k} h_{i,k}(Z_i) z_i - \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_{i,k} \\ & - \tilde{\lambda}_{i,k}\dot{\hat{\lambda}}_{i,k} - \tilde{\tau}_{i,k}\dot{\hat{\tau}}_{i,k} + g_{i,k} z_i (z_{i+1} + \alpha_i) \\ & - g_{i-1,k} z_{i-1} z_i \end{aligned} \tag{22}$$

Based on (22), the virtual control signal α_i is designed in the following form

$$\alpha_i = -\frac{1}{g_m}\left[r_i z_i + K(Z_i)\alpha_i^f + (1 - K(Z_i))\alpha_i^r\right] \tag{23}$$

where $r_i > 0$ is the constant, and $\alpha_i^f = \hat{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}} P_{m_i}(Z_i) + \hat{\tau}_{i,k}$, $\alpha_i^r = \hat{\lambda}_{i,k} h_{i,k}(Z_i)$.

In view of Assumption 3, the virtual control signal α_i and (22), the time derivative of the candidate Lyapunov function V_i can be rewritten as

$$\begin{aligned}\dot{V}_i \le\ & \dot{V}_{i-1} - r_i z_i^2 + g_{i,k} z_i z_{i+1} + K(Z_i) z_i \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}} P_{m_i} \\ & + (1 - K(Z_i)) z_i \tilde{\lambda}_{i,k} h_{i,k} + K(Z_i) z_i \tilde{\tau}_{i,k} \\ & - g_{i-1,k} z_{i-1} z_i - \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}}_{i,k} - \tilde{\lambda}_{i,k}\dot{\hat{\lambda}}_{i,k} - \tilde{\tau}_{i,k}\dot{\hat{\tau}}_{i,k} \end{aligned} \tag{24}$$

With full consideration of (24), three adaptive laws are designed as

$$\begin{cases}\dot{\hat{\boldsymbol{\theta}}}_{i,k} = K(Z_i) z_i P_{m_i} \\ \dot{\hat{\lambda}}_{i,k} = (1 - K(Z_i)) z_i h_{i,k} \\ \dot{\hat{\tau}}_{i,k} = K(Z_i) z_i \end{cases} \tag{25}$$

Following a detailed mathematical derivation, according to (24) and (25), resulting in the following inequality

$$\dot{V}_i \le -\sum_{j=1}^{i} r_j z_j^2 + g_{i,k} z_i z_{i+1} \tag{26}$$

Step n: The candidate Lyapunov function V_n is considered as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{n,k} + \frac{1}{2}\tilde{\lambda}_{n,k}^2 + \frac{1}{2}\tilde{\tau}_{n,k}^2 \tag{27}$$

With the help of (1), the time derivative of the candidate Lyapunov function V_n can be written as

$$\begin{aligned}\dot{V}_n =\ & \dot{V}_{n-1} + \left[g_{n,k} u + F_{n,k}(Z_n)\right] z_n - g_{n-1,k} z_{n-1} z_n \\ & - \tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{n,k} - \tilde{\lambda}_{n,k}\dot{\hat{\lambda}}_{n,k} - \tilde{\tau}_{n,k}\dot{\hat{\tau}}_{n,k} \end{aligned} \tag{28}$$

where $F_{n,k}(Z_n) = f_{n,k} - \dot{\alpha}_{n-1} + g_{n-1,k} z_{n-1}$, $Z_n = \left[\bar{\boldsymbol{x}}_n, \bar{y}_d^{(n)}, \bar{\hat{\boldsymbol{\theta}}}_{n-1,k}, \bar{\hat{\lambda}}_{n-1,k}, \bar{\hat{\tau}}_{n-1,k}\right]^{\mathrm{T}}$.

According to Lemma 2, $F_{n,k}(Z_n)$ as the combination of continuous and unknown nonlinear functions can be estimated by MTN. Namely, for given $\tau_{n,k} > 0$, there is one MTN as $\boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}(Z_n)$, such that

$$F_{n,k}(Z_n) = \boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}(Z_n) + \varepsilon_{n,k}(Z_n), \left|\varepsilon_{n,k}\right| < \tau_{n,k} \tag{29}$$

where $\varepsilon_{n,k}(Z_n)$ defines the estimation error.

In the light of Assumption 2, there exist an unknown constant $\lambda_{n,k}$ and a known and positive function $h_{n,k}(Z_n)$ such that the following inequality is correct

$$\left|F_{n,k}(Z_n)\right| \le \lambda_{n,k} h_{n,k}(Z_n) \tag{30}$$

Combining (28), (29) with (30), the time derivative of the candidate Lyapunov function V_n can be rewritten as

$$\begin{aligned}\dot{V}_n \le\ & g_{n,k} z_n u - g_{n-1,k} z_{n-1} z_n - \tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{n,k} - \tilde{\lambda}_{n,k}\dot{\hat{\lambda}}_{n,k} \\ & - \tilde{\tau}_{n,k}\dot{\hat{\tau}}_{n,k} + (1 - K(Z_n))\lambda_{n,k} h_{n,k}(Z_n) z_n \\ & + K(Z_n)\left(\boldsymbol{\theta}_{n,k}^{\mathrm{T}} P_{m_n}(Z_n) + \tau_{n,k}\right) z_n + \dot{V}_{n-1} \end{aligned} \tag{31}$$

The actual control input u is considered as

$$u = -\frac{1}{g_m}\left[r_n z_n + K(Z_n)\alpha_n^f + (1 - K(Z_n))\alpha_n^r\right] \tag{32}$$

where $r_n > 0$ is a constant, and $\alpha_n^f = \hat{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}} P_{m_n}(Z_n) + \hat{\tau}_{n,k}$, $\alpha_n^r = \hat{\lambda}_{n,k} h_{n,k}(Z_n)$.

On the basis of Assumption 3, the actual control input u and (31), the time derivative of the candidate Lyapunov function V_n can be written as

$$\begin{aligned}\dot{V}_n \le\ & -r_n z_n^2 - g_{n-1,k} z_{n-1} z_n + K(Z_n) z_n \tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}} P_{m_n} \\ & + (1 - K(Z_n)) z_n \tilde{\lambda}_{n,k} h_{n,k} + K(Z_n) z_n \tilde{\tau}_{n,k} \\ & - \tilde{\boldsymbol{\theta}}_{n,k}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_{n,k} - \tilde{\lambda}_{n,k}\dot{\hat{\lambda}}_{n,k} - \tilde{\tau}_{n,k}\dot{\hat{\tau}}_{n,k} + \dot{V}_{n-1} \end{aligned} \tag{33}$$

Therefore, three adaptive laws are designed as

$$\begin{cases}\dot{\hat{\boldsymbol{\theta}}}_{n,k} = K(Z_n) z_n P_{m_n} \\ \dot{\hat{\lambda}}_{n,k} = (1 - K(Z_n)) z_n h_{n,k} \\ \dot{\hat{\tau}}_{n,k} = K(Z_n) z_n \end{cases} \tag{34}$$

Considering the form of (26) for $i = n - 1$ and combining (33) and (34), it follows that

$$\dot{V}_n \le -\sum_{j=1}^{n} r_j z_j^2 \tag{35}$$

Remark 4: In the control signals (14), (23), (32) designed in this paper, α_i^f denotes the traditional control term of dominant approximation, and α_i^r means the extra control term that is used to cope with outside the approximate domain. Since the estimation ability of MTN is only reflected in the compact set, a multi-switching based controller is considered in this paper. When the MTN input leaves the approximation domain, the controller switches to the extra control term, that is, $K(Z_i) = 0$. Therefore, all signals in the system (1) are ensured to be GUUB.

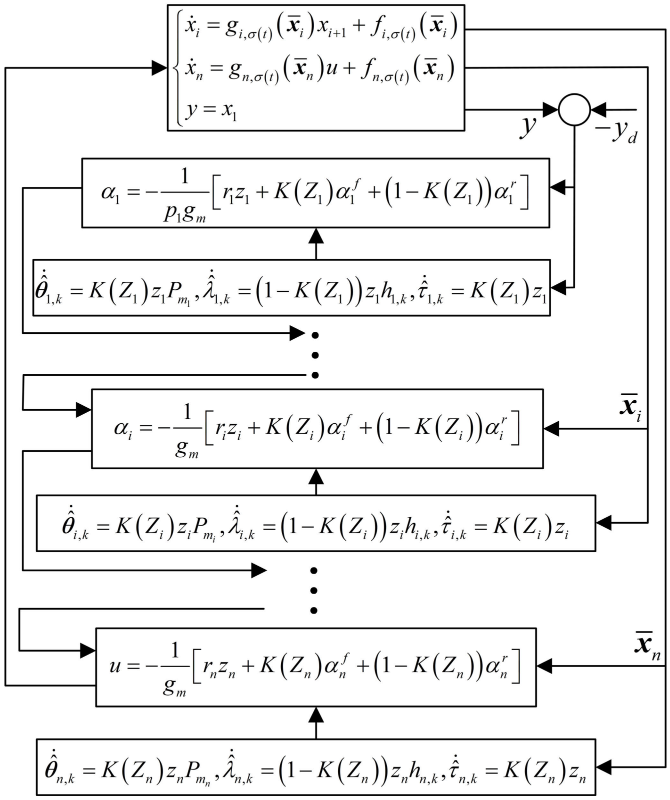

Fig. 2. The block diagram of control process.

IV. Global Stability Analysis

Theorem 1: Based on Assumptions 1-3 and Lemmas 1-2, for the switched nonlinear system (1), the virtual control signals are designed as (14) and (23), actual control input is constructed as (32), the adaptive laws are considered as (16), (25) and (34), for arbitrary initial conditions, the following two points hold:

(i) All of the signals in the closed-loop system remain GUUB;

(ii) The tracking error can approach zero asymptotically and evolve within the prescribed boundary $\wp$.

Proof: The Lyapunov function V is designed as

$$V = \frac{1}{2}\left(\sum_{j=1}^{n} z_j^2 + \sum_{j=1}^{n} \tilde{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}_{j,k} + \sum_{j=1}^{n} \tilde{\lambda}_{j,k}^2 + \sum_{j=1}^{n} \tilde{\tau}_{j,k}^2\right) \tag{36}$$

According to (35), $\dot{V}$ can be written as

$$\dot{V} \leq -\sum_{j=1}^{n} r_j z_j^2 \tag{37}$$

It follows that V is a monotonically decreasing function, and for $i = 1, 2, \cdots, n$, $k \in M$, the signals $z_i, \tilde{\boldsymbol{\theta}}_{i,k}, \tilde{\lambda}_{i,k}, \tilde{\tau}_{i,k}$ in the closed-loop system are bounded. Therefore, in view of $\hat{\boldsymbol{\theta}}_{i,k} = \boldsymbol{\theta}_{i,k} - \tilde{\boldsymbol{\theta}}_{i,k}$, $\hat{\lambda}_{i,k} = \lambda_{i,k} - \tilde{\lambda}_{i,k}$ and $\hat{\tau}_{i,k} = \tau_{i,k} - \tilde{\tau}_{i,k}$, we can get that $\hat{\boldsymbol{\theta}}_{i,k}, \hat{\lambda}_{i,k}, \hat{\tau}_{i,k}$ are bounded. The boundedness of α_i and u can be ensured through the expression of the designed control signals (14), (23) and (32). Based on the coordinate transformations (8), it can be further concluded that x_i is bounded. With the help of Lemma 3, it is used to figure out that globally asymptotically stable can be derived by the multi-switching-based control strategy proposed in this paper.

To sum up, the control framework of this paper can be derived from the designed multi-switching controller, which is shown in Fig. 2.

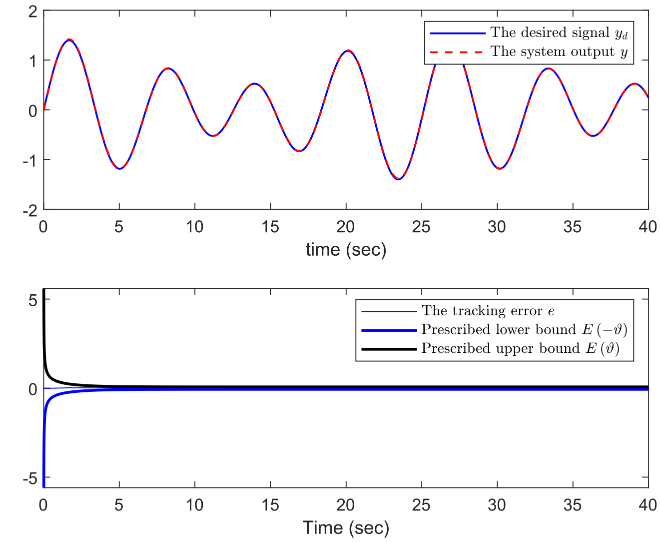

Fig. 3. Desired signal y_d, system output y, tracking error $y - y_d$ and prescribed functions.

V. Simulation Study

Example 1 (Numerical Example): A mathematical example is considered, and its switched nonlinear system model can be described in the following form

$$\begin{cases} \dot{x}_1 = g_{1,\sigma(t)} x_2 + f_{1,\sigma(t)} \\ \dot{x}_2 = g_{2,\sigma(t)} u + f_{2,\sigma(t)} \\ y = x_1 \end{cases} \tag{38}$$

where the switching signal is denoted as $\sigma(t) \in M = \{1, 2\}$, the initial state is considered as $\boldsymbol{x}(\mathbf{0}) = [x_1(0), x_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, the continuous and differentiable functions are designed as $g_{1,1} = g_{1,2} = 1$, $g_{2,1} = 1 + 0.5\cos x_1$, $g_{2,2} = 1 + 0.5\sin x_2$, $f_{1,1} = x_1^2$, $f_{1,2} = x_1 \sin x_1$, $f_{2,1} = x_1 x_2^2$, $f_{2,2} = x_1^2 x_2$. The desired signal is selected as $y_d = 0.95(\sin t + 0.5\sin 0.75t)$.

On the basis of Theorem 1, the adaptive multi-switching-based control structures are designed as

$$\alpha_1 = -\frac{1}{p_1 g_m}\left[r_1 z_1 + K(Z_1)\alpha_1^f + (1 - K(Z_1))\alpha_1^r\right] \tag{39}$$

$$\alpha_i = -\frac{1}{g_m}\left[r_i z_i + K(Z_i)\alpha_i^f + (1 - K(Z_i))\alpha_i^r\right] \tag{40}$$

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_{j,k} = K(Z_j) z_j P_{m_j} \\ \dot{\hat{\lambda}}_{j,k} = (1 - K(Z_j)) z_j h_{j,k} \\ \dot{\hat{\tau}}_{j,k} = K(Z_j) z_j \end{cases} \tag{41}$$

where $i = 2, \cdots, n$, $j = 1, \cdots, n$, $\alpha_j^f = \hat{\boldsymbol{\theta}}_{j,k}^{\mathrm{T}} P_{m_j}(Z_j) + \hat{\tau}_{j,k}$, $\alpha_j^r = \hat{\lambda}_{j,k} h_{j,k}(Z_j)$, $\alpha_n = u$.

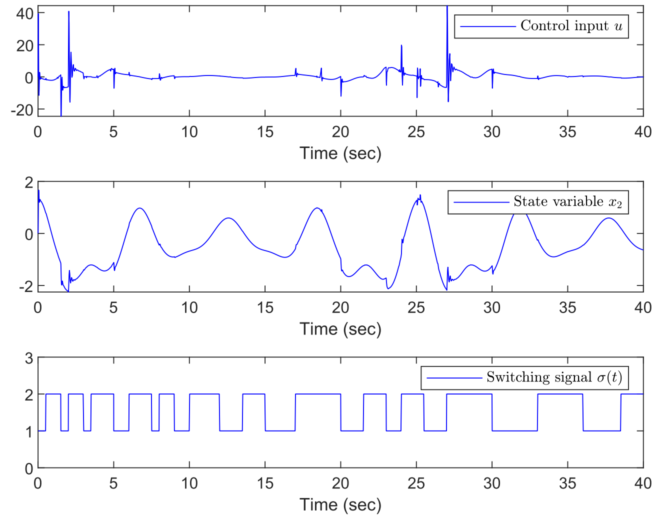

Fig. 4. Control input u, state variable x_2 and switching signal.

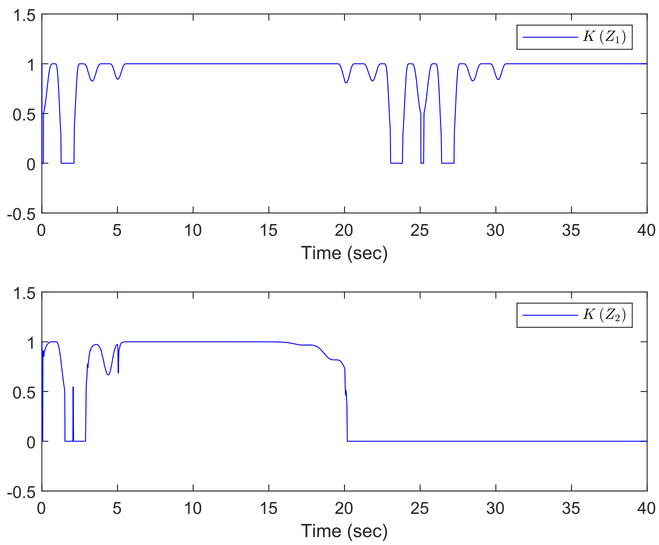

Fig. 5. Multi-switching signals.

The design parameters in the controller are selected as $g_m = 0.5$, $s_1 = 1$, $s_2 = 1.3$, $c = 0.2$, $r_f = 0.15$, $r_1 = 50$, $r_2 = 20$. The time-varying scaling function and the normalized function are designed as $\gamma(t) = \frac{1}{(1-r_f)e^{-0.6t}+r_f}$ and $\varphi(e) = \frac{e}{(e^2+c)^{0.5}}$. The known and positive functions $h_{1,1} = h_{1,2} = 2x_1^2 + \dot{y}_d^2$ and $h_{2,1} = h_{2,2} = x_1^2 + x_2^2 + \dot{y}_d^2 + 1$ are considered. The developed control results of the switched nonlinear system (38) are shown in Figs. 3-5.

Fig. 3 displays the trajectories of desired signal y_d, system output y, the tracking error of the system (38) and the trajectories of prescribed functions, which indicates that superior tracking performance is achieved by the control strategy designed in this paper. The trajectories of control input u, system state x_2 and switching signal $\sigma(t)$ are given in Fig. 4. Fig. 5 depicts multi-switching signals $K(Z_1)$, $K(Z_2)$.

In addition, in order to further demonstrate that the tracking control strategy proposed in this paper is independent of the initial conditions, the tracking errors in five cases ($x_1(0) = 0, 0.5, -0.5, 1, -1$) are shown in Fig. 6.

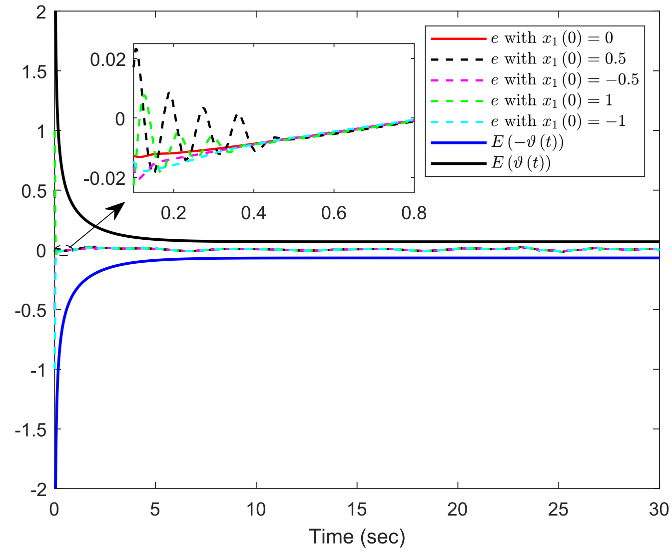

Fig. 6. Tracking performance under multiple initial conditions.

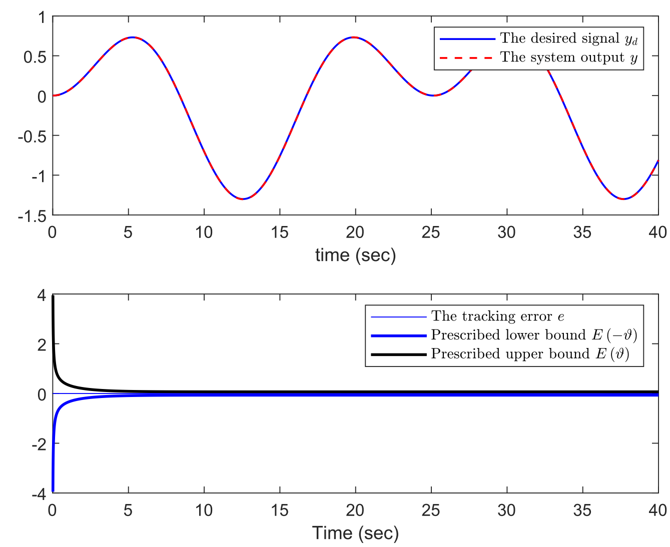

Fig. 7. Desired signal y_d, system output y, tracking error $y - y_d$ and prescribed functions.

It can be found from Fig. 6 that the tracking error can be quickly converge to near the origin and evolve within the prescribed boundary $\wp$. Therefore, it can be concluded from Figs. 3-6 that the control objectives can be realized by the controller proposed in this paper.

Example 2 (Practical Example): A single-link inverted pendulum that can be modeled as the switched nonlinear system is discussed, which verifies the practical application value of the control strategy developed in this paper

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{\cos x_1}{P_{\sigma(t)}} u + \dfrac{\left(m_{\sigma(t)} + m_c\right) g \sin x_1 - Q_{\sigma(t)}}{P_{\sigma(t)}} \\ y = x_1 \end{cases} \tag{42}$$

where $P_{\sigma(t)} = \frac{4}{3} l \left(m_{\sigma(t)} + m_c\right) - m_{\sigma(t)} l \cos^2 x_1$, $Q_{\sigma(t)} = \frac{1}{2} m_{\sigma(t)} l x_2^2 \sin 2x_1$, $\sigma(t) \in M = \{1, 2\}$ indicates the switching signal, $m_{\sigma(t)}$ denotes the mass of the pendulum with

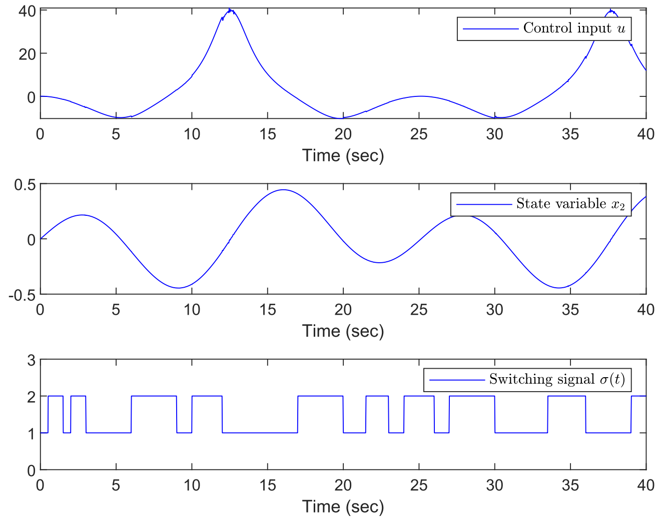

Fig. 8. Control input u, state variable x_2 and switching signal.

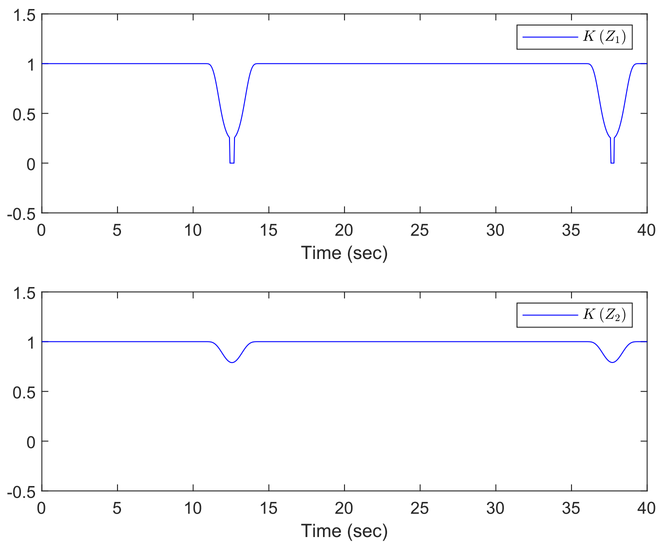

Fig. 9. Multi-switching signals.

$m_1 = 0.1kg$ and $m_2 = 0.15kg$, $m_c = 1kg$ represents the mass of cart, $g = 9.8m/s^2$ indicates the gravitational acceleration, $l = 0.5m$ expresses the length of the pendulum. The desired signal is designed as $y_d = 0.65(\cos 0.25t - \cos 0.5t)$.

The adaptive multi-switching-based control structures, i.e. (39), (40), (41), are employed in Example 2. Among them, $g_m = 0.3$, and other design parameters, initial state $\boldsymbol{x}(\mathbf{0})$, the time-varying scaling function $\gamma(t)$, the normalized function $\varphi(e)$ and positive functions $h_{1,1}, h_{1,2}, h_{2,1}, h_{2,2}$ are the same as those in Example 1. The developed control results of the single-link inverted pendulum system (42) and the tracking errors in five cases $(x_1(0) = 0, 0.5, -0.5, 1, -1)$ are shown in Figs. 7-10.

It can be concluded from Figs. 7-10 that for the practical systems, the controller proposed in this paper not only is independent of the initial state but also achieves excellent tracking performance. Therefore, the effectiveness of the adaptive multi-switching-based global control strategy developed in this paper is further verified.

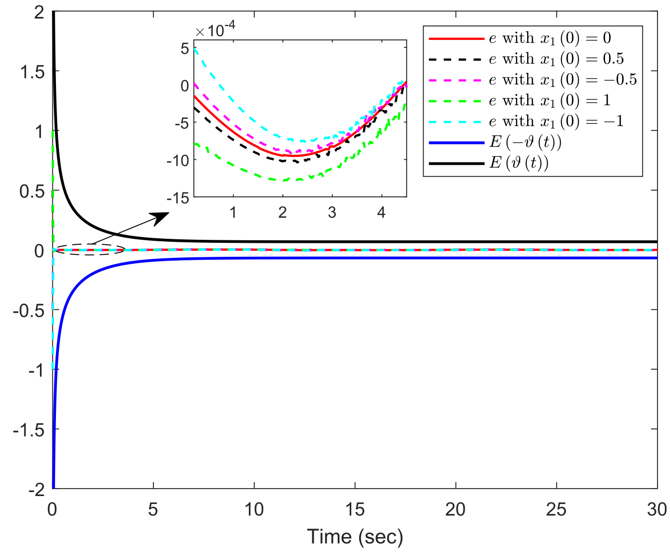

Fig. 10. Tracking performance under multiple initial conditions.

VI. Conclusion

The global prescribed performance control of switched nonlinear systems is researched. First of all, the tracking error constraint problem is transformed into the bounded problem of a new variable by introducing a normalized function and a barrier function. In addition, by exploiting the common Lyapunov function method and multi-dimensional Taylor network (MTN), a multi-switching-based adaptive controller is constructed to realize asymptotic tracking. Finally, the benefits and reasonableness of the proposed control strategy are implied by theoretical analysis and simulations. It is worth noting that the global control is first made using the MTN estimation technique. Further research concerned is how to extend the work of this paper to switched nonlinear systems under multiple faults.

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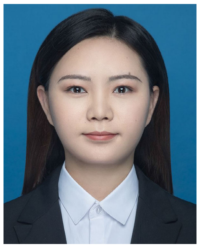

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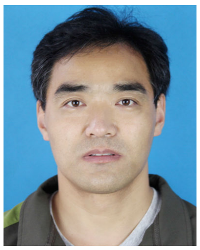

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