Nonlinear Dyn
https://doi.org/10.1007/s11071-025-11850-5

RESEARCH

Adaptive event-triggered control for multi-agent systems with state time-delays and full state constraints

Zhao-Yi Zong · Fen-Fen Guan · Shan-Liang Zhu · Yu-Qun Han

Received: 15 July 2025 / Revised: 15 September 2025 / Accepted: 22 September 2025

Abstract This paper proposes an innovative adaptive event-triggered control strategy for a class of state time-delays nonlinear multi-agent systems (MASs) with full state constraints. Firstly, a performance function is introduced to guarantee the output consensus error remains within a prescribed range. Secondly, by integrating barrier Lyapunov function (BLF) and Lyapunov-Krasovskii (LK) function, a novel Lyapunov function is constructed to ensure that system states satisfy constraint conditions while effectively mitigating the adverse effects of state time-delays. Furthermore, multi-dimensional Taylor networks (MTNs) are employed to approximate unknown nonlinear terms. Thirdly, an event-triggered mechanism is implemented to curtail communication overhead. Theoretical analysis proves that all signals are bounded, the tracking errors fulfill the desired performance requirements in both transient and steady states, and the Zeno phenomenon is excluded. Finally, the effectiveness of the proposed strategy is further validated through three simulation examples.

Keywords Multi-agent systems · State time-delays · Full state constraints · Event-triggered control · Multi-dimensional Taylor networks

Z.-Y. Zong · F.-F. Guan · S.-L. Zhu · Y.-Q. Han (✉)
School of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao 266061, China
e-mail: yuqunhan@qust.edu.cn

1 Introduction

Recently, nonlinear multi-agent systems (MASs) have gained widespread adoption in intelligent manufacturing [1], energy and power systems [2], and robotics [3] due to their significant advantages in cooperative control. Specifically, the leader-follower consensus problem, as a core objective of cooperative control, has attracted substantial research attention and produced a large number of research results [4–7]. However, existing methods generally assume the system dynamics are known or the nonlinear parameters are determined, rendering them inadequate for systems with complex nonlinearities. To address this limitation, researchers have integrated backstepping techniques with neural networks [8,9] and fuzzy logic systems [10–12] to develop control strategies for MASs. Among these approaches, multi-dimensional Taylor network (MTN), a novel type of neural network model, has demonstrated notable advantages in MASs owing to its straightforward architecture and low computational complexity. Additionally, it has also achieved preliminary success [13–15]. It is important to observe that although the above research has made certain progress, it has not fully considered issues such as information transmission delay in actual systems.

It is well known that state time-delays are ubiquitous and non-ignore in physical systems, frequently serving as a critical factor in system instability. Consequently, effectively addressing state time-delays is essential.

Published online: 30 September 2025

Common approaches to this problem primarily fall into three categories: using the current states for predictive control [16], designing time-delays compensation estimators [17,18], and constructing Lyapunov-Krasovskii (LK) function [19–25]. Compared to the first two methods, the latter not only enables systematic derivation of stability criteria but also proves applicable to nonlinear systems [19,20], MASs [21–23], and event-triggered mechanisms [24,25]. Crucially, it does not require precise prediction of time-delays information. However, none of the above studies considered the impact of full state constraints on the system. To meet safety requirements and performance specifications, full state constraints must be explicitly considered. In this context, a number of control strategies have been put forward to tackle full state constraints [26–29]. Nevertheless, in the presence of external disturbances and parameter uncertainties, conventional schemes frequently exhibit limitations in guaranteeing satisfactory dynamic performance throughout the convergence process.

To further enhance system performance in dynamic environment, the prescribed performance control (PPC) method has been proposed. This method utilizes a predefined performance function to prescribe explicit convergence boundaries for the tracking error, ensuring that the error satisfies performance indices. The efficacy of PPC has been demonstrated across diverse systems, including general systems [30,31], stochastic systems [32,33] and MASs [34,35]. Although the PPC method has significant advantages in terms of performance assurance, it may induce high-frequency control updates and elevated communication overhead. To alleviate this issue, the event-triggered control (ETC) strategy was proposed. The authors of [36] were the first to apply the event-triggered mechanism to the consistency problem in MASs. Subsequently, the authors of [37] proposed a distributed ETC strategy to solve the consistency problem in MASs, making pioneering contributions to the application of ETC in MASs. Since then, ETC has rapidly developed in MASs and has achieved numerous results in first-order [38,39], second-order [40,41], and high-order [42,43] systems. However, existing research has neglected the simultaneous consideration of state time-delays, full state constraints, and event-triggered mechanism. The compounded complexity arising from jointly addressing these three factors has significantly hindered controller design, resulting in relatively scarce literature on this integrated problem. Nevertheless, this very challenge presents critical research opportunities and potential avenues for breakthroughs.

Based on the previous analysis, this study investigates the adaptive ETC design for a class of MASs characterized by state time-delays and full-state constraints under a directed communication graph. In contrast to existing literature, the primary contributions of this work are outlined as follows

(1) This paper presents the first study to incorporate MTNs into state time-delayed nonlinear MASs with full state constraints and proposes a new adaptive event-triggered control strategy. The proposed approach guarantees the boundedness of system signals and ensures that the tracking error conforms to the prescribed performance specifications. Compared to conventional control schemes [8–12], the MTN approximation method employed in this scheme effectively simplifies the complexity of controller design through its simple network structure, while concurrently enhancing both practicality and operational efficiency. Furthermore, reference [16], addresses state time-delays using a prediction-based control method. Meanwhile, references [17,18], employ delay compensation estimators to mitigate the effects of state time-delays. Unlike the treatment of state-delays in [16–18], this paper constructs a LK function containing delay terms and combines it with the barrier Lyapunov function, which not only effectively compensates for the adverse effects of state time-delays but also strictly guarantees that all states of the system meet the constraints throughout the operation process.

(2) Although studies [16–28] have addressed the issues of state time-delays and full state constraints, they lack further exploration into the preset performance problem. Similarly, references [30–35] only consider the preset performance problem but ignore the influence of state time-delays and full state constraints on the system. Therefore, this paper introduces a performance function based on a comprehensive consideration of state time-delays and full state constraints, which strictly limits the system error within a preset region. This ensures that the system meets performance indicators such as convergence rate, overshoot range, and steady-state accuracy during the dynamic response process, and comprehensively improves the responsiveness and robustness of the system in dynamic environments.

This scheme takes into account both control performance and safety requirements and has stronger practicality and representativeness.

(3) Compared with [20,26,34], this study introduces an ETC mechanism, which enables a notable reduction in the update frequency of the controller while preserving the convergence performance of the system. It further reduces communication burden and computational cost effectively, which in turn makes the proposed control strategy more advantageous in resource-constrained networked MASs.

2 Problem description and knowledge preparation

2.1 Graph theory

Let $\zeta = (\aleph, \vartheta, A)$ denotes the directed graph, in which $\aleph = \{\upsilon_1, \upsilon_2, \cdots, \upsilon_N\}$ and $\vartheta \subseteq \aleph \times \aleph$ represent the node and edge sets. The edge from node n to m is denoted by $(\upsilon_n, \upsilon_m) \in \vartheta$, indicating that agent m acquires information from agent n. The symbol $A = (a_{m,n})_{N\times N}$ denotes the adjacency matrix. In this matrix, $a_{mn} \geq 0$ implies $(\upsilon_n, \upsilon_m) \in \vartheta$, otherwise $a_{mn} = 0$. The degree matrix is defined as $D = diag\{d_1, d_2, \cdots, d_N\}$ with $d_i = \sum_{j\in N_i} a_{ij}$. The Laplacian matrix of the directed graph $L = D - A$. Define $\mathrm{B} = diag\{b_1, b_2, \cdots, b_N\}$, when the ith node can receive the message from the leader, $b_m = 1$, otherwise, $b_m = 0$.

Assumption 1 [44]: If the directed graph ζ has a spanning tree, that is, there exists a path connects the root node to other nodes, then the matrix $L + \mathrm{B}$ is nonsingular, and the virtual leader's desired trajectory is represented by y_{d}.

2.2 Problem description

Consider the following nonlinear MASs subject to state time-delays

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1} + f_{i,j}(\bar{\boldsymbol{x}}_{i,j}) + \phi_{i,j}(\bar{\boldsymbol{x}}_{i,j}(t-\tau_{i,j})) \\ \quad + \Delta_{i,j}(\bar{\boldsymbol{x}}_{i,j}(t)), \\ \quad 1 \leq j \leq n-1 \\ \dot{x}_{i,n} = u_i + f_{i,n}(\bar{\boldsymbol{x}}_{i,n}) + \phi_{i,n}(\bar{\boldsymbol{x}}_{i,n}(t-\tau_{i,n})) \\ \quad + \Delta_{i,n}(\bar{\boldsymbol{x}}_{i,n}(t)) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $i = 1, \cdots, N$, $\bar{\boldsymbol{x}}_{i,j} = \left[x_{i,1}, \cdots, x_{i,n}\right]^T \in R^n$ and $u_i \in R$ represent the state vector and control input of the ith agent. $f_{i,j}\left(\bar{\boldsymbol{x}}_{i,j}\right)$ and $\phi_{i,j}\left(\bar{\boldsymbol{x}}_{i,j}\left(t-\tau_{i,j}\right)\right)$ denote two smooth nonlinear functions that are unknown in the system. $\tau_{i,j}$ represents the unknown state time-delays. To simplify the notation, let $t - \tau_{i,j} \triangleq t^{\tau}_{i,j}$, hence, $\bar{\boldsymbol{x}}_{i,j}\left(t-\tau_{i,j}\right)$ can be simplified as $\bar{\boldsymbol{x}}_{i,j}\left(t^{\tau}_{i,j}\right)$. $\Delta_{i,j}\left(\bar{\boldsymbol{x}}_{i,j}(t)\right)$ denote the external disturbances of MASs.

Remark 1 This paper investigates nonlinear MASs with state time-delays and external disturbances. It is worth noting that many practical systems can be modeled by system (1), including robotic and manipulator systems, mass–spring–damper systems, as well as marine vessels and underwater robotic platforms.

For the MASs (1), the purpose of this paper is to design an event-triggered-based adaptive control strategy to achieve semi-global bounded of the system, and fulfill the following objectives

i) every agent achieves tracking of the expected trajectory and the output consensus error satisfies predetermined performance.
ii) states $x_{i,j}(t)$ of the system are satisfied $\left|x_{i,j}(t)\right| < \kappa_{i,j}(t)$, $j = 1, 2, \cdots, n$, where $\kappa_{i,j}(t) > 0$ are known continuous functions.
iii) effectively reduce communication burden without Zeno behavior occurring.

To simplify the process of designing the control scheme, several assumptions and lemmas are presented below

Assumption 2 [45]: All time derivatives of the signal y_d up to the nth order are bounded and continuous. i.e. they satisfy the conditions $|y_{\mathrm{d}}| < \Gamma_1$ and $\left|y_d^{(l)}\right| < \Gamma_{l1} (l = 2, \ldots, n)$, where $\Gamma_1 > 0$ and $\Gamma_{l1} > 0$.

Assumption 3 There exist constants $\ell_{i,j} > 0$ such that external disturbance $\left|\Delta_{i,j}\left(\bar{\boldsymbol{x}}_{i,j}(t)\right)\right| \leq \ell_{i,j}$.

Assumption 4 It is assumed that the state time-delays function satisfies $\left|\phi_{i,j}\left(\bar{\boldsymbol{x}}_{i,j}\right)\right| \leq \sum_{l=1}^{j} \varphi_{i,j,l}\left(x_{i,j}\right)$ with $\varphi_{i,j,l}\left(x_{i,j}\right)$ are unknown positive functions.

Lemma 1 [46]: *Y_d is N-dimensional vector that satisfies $\|Y - Y_d\| \leq \frac{\|\boldsymbol{e}_1\|}{\sigma_{\min}}$, where $\boldsymbol{e}_1 = \left[z_{1,1}, z_{2,1}, \ldots, z_{N,1}\right]^T \in R^N$, $Y = [y_1, y_2, \cdots, y_N]^T \in R^N$, $Y_d =$*

$[y_0, y_0, \cdots, y_0]^T \in R^N$, *$\sigma_{\min}$ is the smallest singular value of matrix* $L + \mathrm{B}$.

Lemma 2 [20]: *Let* $\Omega_{z_{i,l}} = \{z_{i,l} \, \big| |z_{i,l}| < 0.8814\delta_{i,l}\}$, $(i = 1, ..., N, l = 1, ..., n)$ *be the defined compact set. If* $z_{i,l} \in \Omega_{z_{i,l}}$, *then* $1 - 2\tanh^2\left(\frac{z_{i,l}}{\delta_{i,l}}\right) > 0$*; otherwise,* $1 - 2\tanh^2\left(\frac{z_{i,l}}{\delta_{i,l}}\right) \le 0$, *where* $\delta_{i,l} > 0$ *is a constant.*

Lemma 3 [47]: *For any* $k_{bp} > 0$, *If the inequality* $|z_{i,p}| < k_{bp}$ *is fulfilled, it can be deduced that*

$$\log\left(\frac{k_{bp}^2}{k_{bp}^2 - z_{i,p}^2}\right) < \frac{z_{i,p}^2}{k_{bp}^2 - z_{i,p}^2}$$

Lemma 4 [48]: *The inequality* $0 \le |\tau| - \tau \tanh\left(\frac{\tau}{\rho}\right) < 0.2785\tau$ *holds for any* $\rho > 0$ *and* $\tau \in R$.

Lemma 5 [49]: *(Young's inequality) For* $\forall(x, y) \in R^n$, *The following inequality holds*

$$xy \le \frac{p^a}{a}|x|^a + \frac{1}{bq^b}|y|^b$$

where $p > 0$, $a > 1$, $b > 1$, $(a - 1)(b - 1) = 1$.

2.3 Multi-dimensional Taylor network

Notably, in this paper, the MTN is employed to precisely approximate the unknown nonlinear terms that surface during the controller design process, thereby enhancing the accuracy and effectiveness of the controller. The structure of the MTN is illustrated in Fig. 1.

Lemma 6 [50]: *On a compact set* Ω, *for a continuous nonlinear function* $F(\mathbf{Z}) : R^n \to R$, *there exists a MTN* $\mathbf{W}^T \mathbf{P}_{m_n}(\mathbf{Z})$, *as described below*

$$F(\mathbf{Z}) = \mathbf{W}^T \mathbf{P}_{m_n}(\mathbf{Z}) + E(\mathbf{Z})$$

where $\mathbf{Z} = [z_1, z_2, \cdots, z_n]^T \in R^n$ *and* $\mathbf{W} = [W_1, W_2, \ldots W_l]^T \in R^l$ *denote the input vector and the weight vector of the MTN, respectively.* $\mathbf{P}_{m_n}(\mathbf{Z}) = \left[z_1, \cdots, z_n, z_1^2, \cdots, z_n^2, \cdots, z_1^m, \cdots, z_n^m\right]^T \in R^l$ *is the middle layer vector of the MTN.* $E(\mathbf{Z})$ *represents the approximate error between* $F(\mathbf{Z})$ *and* $\mathbf{W}^T \mathbf{P}_{m_n}(\mathbf{Z})$. $|E(\mathbf{Z})| < \varepsilon$ *with* $\varepsilon > 0$, *and* $\mathbf{W} := \arg\min_{W \in R^l}\left\{\sup_{z \in \Omega}\left|F(\mathbf{Z}) - \mathbf{W}^T \mathbf{P} m_n(\mathbf{Z})\right|\right\} \in R^l$.

Remark 2 It is important to emphasize that while the MTN bears structural similarity to the radial basis function neural network (RBFNN), a core distinction exists in the design of their middle layers. Specifically, the RBFNN relies on Gaussian basis functions to achieve nonlinear mapping, whereas the MTN adopts polynomial approximation for capturing nonlinear characteristics. This substitution not only simplifies the overall network structure but also significantly decreases computational overhead.

3 Main results

3.1 Coordinate transformation

The output consensus error e_i is designed as follows

$$e_i(t) = \sum_{j \in N_i} a_{ij}\left(y_i - y_j\right) + b_i(y_i - y_0) \tag{2}$$

And e_i satisfies the following predetermined performance requirements

$$-l_m\psi(t) \le e_i(t) \le l_M\psi(t) \tag{3}$$

where the performance function $\psi(t)$ is strictly decreasing and bounded, and is given by $\psi(t) = (\psi_0 - \psi_\infty)e^{-vt} + \psi_\infty$. l_m and l_M are tunable design parameters. $\psi_0 = \psi(0)$, $\psi_0 > \psi_\infty > 0$ and $v > 0$. The initial value of the error $e_i(0)$ meets $-l_m\psi(0) \le e_i(0) \le l_M\psi(0)$.

Next, the following equivalent transformation is employed to satisfy the prescribed performance requirement.

$$e_i(t) = \psi(t) H_i(w_i(t)), t \ge 0 \tag{4}$$

where $H_i(w_i(t)) = \frac{l_M e^{w_i} - l_m e^{-w_i}}{e^{w_i} + e^{-w_i}}$ with $w_i(t)$ denotes the transformation error.

Based on the above discussion, define the following coordinate transformation

$$\begin{cases} z_{i,1} = w_i - \dfrac{1}{2}\ln\left(\dfrac{l_m}{l_M}\right) \\ z_{i,j} = x_{i,j} - \alpha_{i,j-1}, \quad 2 \le j \le n \end{cases} \tag{5}$$

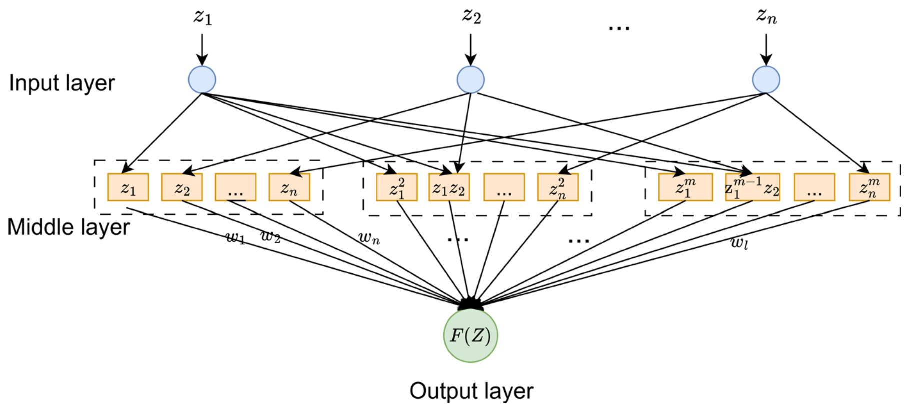


Fig. 1 Structural representation of MTN

Remark 3 Given that the function $H_i(w_i(t))$ is strictly increasing and $\frac{\partial H_i}{\partial w_i} = \frac{2(l_M+l_m)}{(e^{w_i}+e^{-w_i})^2} > 0$, it follows that $w_i(t) = H_i^{-1}\left(\frac{e_i(t)}{\psi_i(t)}\right) = \frac{1}{2}\ln\left(\frac{H_i+l_m}{l_M-H_i}\right)$. By differentiating this relation, we obtain $\dot{w}_i(t) = r_i\left(\dot{e}_i - \frac{\dot{\psi}e_i}{\psi}\right)$, under the condition that $r_i = \frac{1}{2\psi}\left[\frac{1}{H_i+l_m} - \frac{1}{H_i-l_M}\right]$ holds. Hence, we can derive $\dot{z}_{i,1} = r_i\left(\dot{e}_i - \frac{\dot{\psi}e_i}{\psi}\right)$.

3.2 Controller design

Step 1: Design the candidate Lyapunov function $\Xi_{i,1}$ as follows

$$\Xi_{i,1} = \frac{1}{2}\ln\frac{k_{b1}^2}{k_{b1}^2 - z_{i,1}^2} + \frac{1}{2}\tilde{\boldsymbol{W}}_{i,1}^T\tilde{\boldsymbol{W}}_{i,1} + \frac{1}{2}\int_{t-\tau_{i,1}}^{t}\varphi_{i,1,1}^2\left(x_{i,1}(s)\right)ds + \frac{1}{2}\sum_{j\in N_i}\int_{t-\tau_{j,1}}^{t}\varphi_{j,1,1}^2\left(x_{j,1}(s)\right)ds \tag{6}$$

where $\tilde{\boldsymbol{W}}_{i,1} = \boldsymbol{W}_{i,1} - \hat{\boldsymbol{W}}_{i,1}$ is estimation error, $\hat{\boldsymbol{W}}_{i,1}$ is the estimate of $\boldsymbol{W}_{i,1}$ and $k_{b1} = \kappa_{i,1} - \iota_{i,1}$ with $\iota_{i,1}$ is a constant.

Calculating the derivative of $\Xi_{i,1}$ as follows

$$\dot{\Xi}_{i,1} = \frac{z_{i,1}}{k_{b1}^2 - z_{i,1}^2}\left(r_i\left(\dot{e}_i - \frac{\dot{\psi}e_i}{\psi}\right) - \frac{z_{i,1}\dot{k}_{b1}}{k_{b1}}\right) - \tilde{\boldsymbol{W}}_{i,1}^T\dot{\hat{\boldsymbol{W}}}_{i,1} + \frac{1}{2}\left(\varphi_{i,1,1}^2\left(x_{i,1}(t)\right) - \varphi_{i,1,1}^2\left(x_{i,1}\left(t_{i,1}^{\tau}\right)\right)\right) + \frac{1}{2}\sum_{j\in N_i}\left(\varphi_{j,1,1}^2\left(x_{j,1}(t)\right) - \varphi_{j,1,1}^2\left(x_{j,1}\left(t_{j,1}^{\tau}\right)\right)\right) \tag{7}$$

Differentiating the tracking error e_i yields

$$\dot{e}_i - \frac{\dot{\psi}e_i}{\psi} = (b_i + d_i)\left(z_{i,2} + \alpha_{i,1} + f_{i,1} + \phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\left(t_{i,j}^{\tau}\right)\right) + \Delta_{i,1}\right) - \sum_{j\in N_i}a_{ij}\left(x_{j,2} + f_{j,1} + \phi_{j,1}\left(\bar{\boldsymbol{x}}_{j,1}\left(t_{j,1}^{\tau}\right)\right) + \Delta_{j,1}\right) - b_i\dot{y}_0 - \frac{\dot{\psi}e_i}{\psi} \tag{8}$$

By substituting (8) into (7), we obtain

$$\dot{\Xi}_{i,1} = \frac{z_{i,1}r_i(b_i + d_i)}{k_{b1}^2 - z_{i,1}^2}\left(z_{i,2} + \alpha_{i,1} + f_{i,1} + \phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\left(t_{i,j}^{\tau}\right)\right) + \Delta_{i,1}\right) - \frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}\sum_{j\in N_i}a_{ij}\left(x_{j,2} + f_{j,1} + \phi_{j,1}\left(\bar{\boldsymbol{x}}_{j,1}\left(t_{j,1}^{\tau}\right)\right) + \Delta_{j,1}\right) - \frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}b_i\dot{y}_0 - \frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}\frac{\dot{\psi}e_i}{\psi} - \frac{z_{i,1}^2\dot{k}_{b1}}{k_{b1}\left(k_{b1}^2 - z_{i,1}^2\right)} - \tilde{\boldsymbol{W}}_{i,1}^T\dot{\hat{\boldsymbol{W}}}_{i,1} + \frac{1}{2}\left(\varphi_{i,1,1}^2\left(x_{i,1}(t)\right) - \varphi_{i,1,1}^2\left(x_{i,1}\left(t_{i,1}^{\tau}\right)\right)\right) + \frac{1}{2}\sum_{j\in N_i}\left(\varphi_{j,1,1}^2\left(x_{j,1}(t)\right) - \varphi_{j,1,1}^2\left(x_{j,1}\left(t_{j,1}^{\tau}\right)\right)\right) \tag{9}$$

According to Lemma 5, Assumption 3 and 4, the following inequalities hold

$$\frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}\phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\left(t_{i,j}^{\tau}\right)\right) \le \frac{1}{2}\frac{(b_i + d_i)z_{i,1}^2r_i^2}{\left(k_{b1}^2 - z_{i,1}^2\right)^2}$$

$$+\frac{\varphi_{i,1,1}^{2}\left(x_{i,1}\left(t_{i,1}^{\tau}\right)\right)}{2\left(b_{i}+d_{i}\right)} \tag{10}$$

$$\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\Delta_{i,1}\leq\frac{1}{2}\frac{\left(b_{i}+d_{i}\right)z_{i,1}^{2}r_{i}^{2}}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)^{2}}+\frac{1}{2\left(b_{i}+d_{i}\right)}\ell_{i,1}^{2} \tag{11}$$

$$-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\phi_{j,1}\left(\bar{\boldsymbol{x}}_{j,1}\left(t_{j,1}^{\tau}\right)\right)\leq\frac{1}{2}\frac{z_{i,1}^{2}r_{i}^{2}}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)^{2}}+\frac{1}{2}\varphi_{j,1,1}^{2}\left(x_{j,1}\left(t_{j,1}^{\tau}\right)\right) \tag{12}$$

$$-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\Delta_{j,1}\leq\frac{1}{2}\frac{z_{i,1}^{2}r_{i}^{2}}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)^{2}}+\frac{1}{2}\ell_{j,1}^{2} \tag{13}$$

Then, by substituting (10), (11), (12) and (13) into (9), we obtain

$$\begin{aligned}\dot{\Xi}_{i,1}\leq&\frac{z_{i,1}r_{i}\left(b_{i}+d_{i}\right)}{k_{b1}^{2}-z_{i,1}^{2}}\left(z_{i,2}+\alpha_{i,1}+f_{i,1}\right)\\&-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\sum_{j\in N_{i}}a_{ij}\left(x_{j,2}+f_{j,1}\right)\\&-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}b_{i}\dot{y}_{0}-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\frac{\dot{\psi}e_{i}}{\psi}-\frac{z_{i,1}^{2}\dot{k}_{b1}}{k_{b1}\left(k_{b1}^{2}-z_{i,1}^{2}\right)}\\&+\frac{\left(b_{i}+d_{i}\right)^{2}z_{i,1}^{2}r_{i}^{2}}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)^{2}}+\sum_{j\in N_{i}}a_{ij}\frac{z_{i,1}^{2}r_{i}^{2}}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)^{2}}\\&-\tilde{\boldsymbol{W}}_{i,1}^{T}\dot{\hat{\boldsymbol{W}}}_{i,1}\\&+\frac{1}{2}\varphi_{i,1,1}^{2}\left(x_{i,1}(t)\right)+\frac{1}{2}\sum_{j\in N_{i}}\varphi_{j,1,1}^{2}\left(x_{j,1}(t)\right)\\&+\frac{1}{2}\ell_{i,1}^{2}+\frac{1}{2}\sum_{j\in N_{i}}a_{ij}\ell_{j,1}^{2}\end{aligned} \tag{14}$$

After adding and subtracting $\tanh^{2}\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\cdot\left(\varphi_{i,1,1}^{2}\left(x_{i,1}(t)\right)+\sum_{j\in N_{i}}\varphi_{j,1,1}^{2}\left(x_{j,1}(t)\right)\right)$, the (14) simplifies to

$$\begin{aligned}\dot{\Xi}_{i,1}\leq&\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\left(\left(b_{i}+d_{i}\right)\left(z_{i,2}+\alpha_{i,1}+F_{i,1}\right)\right.\\&\left.-\frac{\left(b_{i}+d_{i}\right)^{2}z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\right)\\&-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\left(\sum_{j\in N_{i}}a_{ij}x_{j,2}+b_{i}\dot{y}_{0}+\frac{\dot{\psi}e_{i}}{\psi}\right)\\&-\frac{z_{i,1}^{2}\dot{k}_{b1}}{k_{b1}\left(k_{b1}^{2}-z_{i,1}^{2}\right)}\\&+\frac{1}{2}\left(1-2\tanh^{2}\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\right)\\&\left(\varphi_{i,1,1}^{2}\left(x_{i,1}(t)\right)+\sum_{j\in N_{i}}\varphi_{j,1,1}^{2}\left(x_{j,1}(t)\right)\right)\\&-\tilde{\boldsymbol{W}}_{i,1}^{T}\dot{\hat{\boldsymbol{W}}}_{i,1}+\frac{1}{2}\ell_{i,1}^{2}+\frac{1}{2}\sum_{j\in N_{i}}a_{ij}\ell_{j,1}^{2}\end{aligned} \tag{15}$$

where $F_{i,1}\left(\boldsymbol{Z}_{i,1}\right)=f_{i,1}+\frac{(b_i+d_i)z_{i,1}r_i}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)}+\frac{z_{i,1}r_i}{\left(k_{b1}^{2}-z_{i,1}^{2}\right)(b_i+d_i)}\sum_{j\in N_{i}}a_{ij}+\frac{(b_i+d_i)z_{i,1}r_i}{k_{b1}^{2}-z_{i,1}^{2}}+\frac{k_{b1}^{2}-z_{i,1}^{2}}{(b_i+d_i)r_iz_{i,1}}\tanh^{2}\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\left(\varphi_{i,1,1}^{2}\left(x_{i,1}(t)\right)+\sum_{j\in N_{i}}\varphi_{j,1,1}^{2}\left(x_{j,1}(t)\right)\right)-\frac{1}{(b_i+d_i)}\sum_{j\in N_{i}}a_{ij}f_{j,1}$ with $\boldsymbol{Z}_{i,1}=\left[z_{i,1},z_{j,1}\right]^{T}$. According to Lemma 6, we obtain

$$F_{i,1}\left(\boldsymbol{Z}_{i,1}\right)=\boldsymbol{W}_{i,1}^{T}\boldsymbol{P}_{m_{i,1}}\left(\boldsymbol{Z}_{i,1}\right)+E_{i,1}\left(\boldsymbol{Z}_{i,1}\right),\ \left|E_{i,1}\left(\boldsymbol{Z}_{i,1}\right)\right|\leq\varepsilon_{i,1} \tag{16}$$

where $\varepsilon_{i,1}>0$ is a constant.

Substituting (16) into (15) yields

$$\begin{aligned}\dot{\Xi}_{i,1}\leq&\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\left(\left(b_{i}+d_{i}\right)\left(z_{i,2}+\alpha_{i,1}+\boldsymbol{W}_{i,1}^{T}\boldsymbol{P}_{m_{i,1}}+E_{i,1}\right)\right.\\&\left.-\frac{\left(b_{i}+d_{i}\right)^{2}z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\right)\\&-\frac{z_{i,1}r_{i}}{k_{b1}^{2}-z_{i,1}^{2}}\left(\sum_{j\in N_{i}}a_{ij}x_{j,2}+b_{i}\dot{y}_{0}+\frac{\dot{\psi}e_{i}}{\psi}\right)\\&-\frac{z_{i,1}^{2}\dot{k}_{b1}}{k_{b1}\left(k_{b1}^{2}-z_{i,1}^{2}\right)}\\&+\frac{1}{2}\left(1-2\tanh^{2}\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\right)\left(\varphi_{i,1,1}^{2}\left(x_{i,1}(t)\right)\right.\\&\left.+\sum_{j\in N_{i}}\varphi_{j,1,1}^{2}\left(x_{j,1}(t)\right)\right)\\&-\tilde{\boldsymbol{W}}_{i,1}^{T}\dot{\hat{\boldsymbol{W}}}_{i,1}+\frac{1}{2}\ell_{i,1}^{2}+\frac{1}{2}\sum_{j\in N_{i}}a_{ij}\ell_{j,1}^{2}\end{aligned} \tag{17}$$

In addition, we have

$$\frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}E_{i,1} \le \frac{1}{2}\frac{(b_i+d_i)\,z_{i,1}^2 r_i^2}{\left(k_{b1}^2 - z_{i,1}^2\right)^2} + \frac{1}{2\,(b_i+d_i)}\varepsilon_{i,1}^2 \tag{18}$$

By substituting (18) into (17), we obtain

$$\begin{aligned}\dot{\Xi}_{i,1} \le{}& \frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}\left((b_i+d_i)\left(z_{i,2}+\alpha_{i,1}+\boldsymbol{W}_{i,1}^T\boldsymbol{P}_{m_{i,1}}\right)\right.\\ &\left.-\frac{(b_i+d_i)^2 z_{i,1}r_i}{2\left(k_{b1}^2 - z_{i,1}^2\right)}\right)\\ &-\frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}\left(\sum_{j\in N_i}a_{ij}x_{j,2}+b_i\dot{y}_0+\frac{\dot{\psi}e_i}{\psi}\right)\\ &-\frac{z_{i,1}^2\dot{k}_{b1}}{k_{b1}\left(k_{b1}^2 - z_{i,1}^2\right)}\\ &+\frac{1}{2}\left(1-2\tanh^2\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\right)\left(\varphi_{i,1,1}^2\left(x_{i,1}(t)\right)\right.\\ &\left.+\sum_{j\in N_i}\varphi_{j,1,1}^2\left(x_{j,1}(t)\right)\right)\\ &-\tilde{\boldsymbol{W}}_{i,1}^T\dot{\hat{\boldsymbol{W}}}_{i,1}+\frac{1}{2}\ell_{i,1}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\varepsilon_{i,1}^2\end{aligned} \tag{19}$$

Accordingly, the first virtual controller $\alpha_{i,1}$ and adaptive law $\dot{\hat{\boldsymbol{W}}}_{i,1}$ are designed as follows

$$\begin{aligned}\alpha_{i,1} ={}& -\hat{\boldsymbol{W}}_{i,1}^T\boldsymbol{P}_{m_{i,1}} - \frac{k_{i,1}z_{i,1}}{r_i\,(b_i+d_i)} - \frac{\bar{\eta}_{i,1}z_{i,1}}{r_i\,(b_i+d_i)}\\ &+\frac{1}{(b_i+d_i)}\left(\sum_{j\in N_i}a_{ij}x_{j,2}+b_i\dot{y}_0+\frac{\dot{\psi}e_i}{\psi}\right)\end{aligned} \tag{20}$$

$$\dot{\hat{\boldsymbol{W}}}_{i,1} = \frac{z_{i,1}r_i}{k_{b1}^2 - z_{i,1}^2}(b_i+d_i)\,\boldsymbol{P}_{m_{i,1}} - \gamma_{i,1}\hat{\boldsymbol{W}}_{i,1} \tag{21}$$

where constants $k_{i,1} > 0$, $\gamma_{i,1} > 0$, $\bar{\eta}_{i,1} = \sqrt{\left(\frac{\dot{k}_{b1}}{k_{b1}}\right)^2 + \eta_{i,1}}$. Obviously, $\bar{\eta}_{i,1} + \frac{\dot{k}_{b1}}{k_{b1}} \ge 0$, therefore, $-\frac{z_{i,1}^2}{k_{b1}^2 - z_{i,1}^2}\left(\bar{\eta}_{i,1} + \frac{\dot{k}_{b1}}{k_{b1}}\right) \le 0$.

Substituting (20) and (21) into (19), we have

$$\begin{aligned}\dot{\Xi}_{i,1} \le{}& -\frac{k_{i,1}z_{i,1}^2}{k_{b1}^2 - z_{i,1}^2} + (b_i+d_i)\,r_i\frac{z_{i,1}z_{i,2}}{k_{b1}^2 - z_{i,1}^2}\\ &+\gamma_{i,1}\tilde{\boldsymbol{W}}_{i,1}^T\hat{\boldsymbol{W}}_{i,1}\\ &+\frac{1}{2}\left(1-2\tanh^2\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\right)\\ &\left(\varphi_{i,1,1}^2\left(x_{i,1}(t)\right)+\sum_{j\in N_i}\varphi_{j,1,1}^2\left(x_{j,1}(t)\right)\right)\\ &+\frac{1}{2}\ell_{i,1}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\varepsilon_{i,1}^2\end{aligned} \tag{22}$$

Step 2: Design the candidate Lyapunov function $\Xi_{i,2}$ as follows

$$\begin{aligned}\Xi_{i,2} ={}& \Xi_{i,1} + \frac{1}{2}\ln\frac{k_{b2}^2}{k_{b2}^2 - z_{i,2}^2} + \frac{1}{2}\tilde{\boldsymbol{W}}_{i,2}^T\tilde{\boldsymbol{W}}_{i,2}\\ &+\frac{1}{2}\sum_{k=1}^{2}\sum_{l=1}^{k}\int_{t-\tau_{i,k}}^{t}\varphi_{i,k,l}^2\left(x_{i,l}(s)\right)ds\\ &+\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{2}\sum_{l=1}^{k}\int_{t-\tau_{j,k}}^{t}\varphi_{j,k,l}^2\left(x_{j,l}(s)\right)ds\end{aligned} \tag{23}$$

where $\tilde{\boldsymbol{W}}_{i,2} = \boldsymbol{W}_{i,2} - \hat{\boldsymbol{W}}_{i,2}$ is estimation error, $\hat{\boldsymbol{W}}_{i,2}$ is the estimate of $\boldsymbol{W}_{i,2}$ and $k_{b2} = \kappa_{i,2} - \iota_{i,2}$ with $\iota_{i,2}$ is a constant.

Calculating the derivative of $\Xi_{i,2}$, and incorporating (1) and (5), we obtain

$$\begin{aligned}\dot{\Xi}_{i,2} ={}& \dot{\Xi}_{i,1} + \frac{z_{i,2}}{\left(k_{b2}^2 - z_{i,2}^2\right)}\left(x_{i,3}+f_{i,2}+\phi_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\left(t_{i,2}^{\tau}\right)\right)\right.\\ &\left.+\Delta_{i,2}-\dot{\alpha}_{i,1}-\frac{z_{i,2}\dot{k}_{b2}}{k_{b2}}\right)\\ &-\tilde{\boldsymbol{W}}_{i,2}^T\dot{\hat{\boldsymbol{W}}}_{i,2}+\frac{1}{2}\sum_{k=1}^{2}\sum_{l=1}^{k}\left[\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)\right.\\ &\left.-\varphi_{i,k,l}^2\left(x_{i,l}\left(t_{i,k}^{\tau}\right)\right)\right]\\ &+\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{2}\sum_{l=1}^{k}\left[\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)-\varphi_{j,k,l}^2\left(x_{j,l}\left(t_{j,k}^{\tau}\right)\right)\right]\end{aligned} \tag{24}$$

where $\dot{\alpha}_{i,1} = \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\left(x_{i,2}+f_{i,1}+\phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\left(t_{i,1}^{\tau}\right)\right)\right.$ $\left.+\Delta_{i,1}\right) + \frac{\partial\alpha_{i,1}}{\partial\hat{\boldsymbol{W}}_{i,1}}\dot{\hat{\boldsymbol{W}}}_{i,1} + \sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial y_0^{(k)}}y_0^{(k+1)} + \frac{\partial\alpha_{i,1}}{\partial r_i}\dot{r}_i +$

$$\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\left(x_{j,k+1}+f_{j,k}+\phi_{j,k}\left(\bar{\boldsymbol{x}}_{j,k}\left(t_{j,k}^{\tau}\right)\right)\right.$$
$$\left.+\Delta_{j,k}\right)+\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial\psi^{(k)}}\psi^{(k+1)}.$$

Based on Lemma 5, Assumptions 3 and 4, we obtain

$$\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\phi_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\left(t_{i,2}^{\tau}\right)\right)\le\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}+\frac{1}{2}\sum_{l=1}^{2}\varphi_{i,2,l}^2\left(x_{i,l}\right)\tag{25}$$

$$\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\Delta_{i,2}\le\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}+\frac{1}{2}\ell_{i,2}^2\tag{26}$$

$$-\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\phi_{i,1}\left(x_{i,1}\left(t_{i,1}^{\tau}\right)\right)\le\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2+\frac{1}{2}\varphi_{i,1,1}^2\left(x_{i,1}\right)\tag{27}$$

$$-\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\Delta_{i,1}\le\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2+\frac{1}{2}\ell_{i,1}^2\tag{28}$$

$$-\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\phi_{j,k}\left(\bar{\boldsymbol{x}}_{j,k}\left(t_{j,k}^{\tau}\right)\right)\le\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}\sum_{k=1}^{2}\sum_{j\in N_i}\left(\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\right)^2+\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\sum_{l=1}^{k}\varphi_{j,k,l}^2\left(x_{j,l}\left(t_{j,k}^{\tau}\right)\right)\tag{29}$$

$$\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\Delta_{j,k}\le\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\left(\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}\left(\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\right)^2\right)+\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\ell_{j,k}^2\tag{30}$$

Substituting (5), (22), (25), (26), (27), (28), (29) and (30) into (24), we have

$$\begin{aligned}\dot{\Xi}_{i,2}=&-\frac{k_{i,1}z_{i,1}^2}{k_{b1}^2-z_{i,1}^2}+\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\left(z_{i,3}+\alpha_{i,2}+F_{i,2}\right)\\&-\frac{z_{i,2}^2\dot{k}_{b2}}{k_{b2}\left(k_{b2}^2-z_{i,2}^2\right)}\\&-\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}+\gamma_{i,1}\tilde{\boldsymbol{W}}_{i,1}^T\dot{\hat{\boldsymbol{W}}}_{i,1}-\tilde{\boldsymbol{W}}_{i,2}^T\dot{\hat{\boldsymbol{W}}}_{i,2}\\&+\frac{1}{2}\left(1-2\tanh^2\left(\frac{z_{i,2}}{\delta_{i,2}}\right)\right)\sum_{k=1}^{2}\sum_{l=1}^{k}\\&\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right)\\&+\frac{1}{2}\left(1-2\tanh^2\left(\frac{z_{i,1}}{\delta_{i,1}}\right)\right)\\&\left(\varphi_{i,1,1}^2\left(x_{i,1}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,1,1}^2\left(x_{j,1}\left(t\right)\right)\right)\\&+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\varepsilon_{i,1}^2+\frac{1}{2}\ell_{i,1}^2+\frac{1}{2}\ell_{i,1}^2\\&+\frac{1}{2}\ell_{i,2}^2+\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\ell_{j,k}^2\end{aligned}\tag{31}$$

where $F_{i,2}=f_{i,2}-\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\left(x_{i,2}+f_{i,1}\right)+\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\left(1+\left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2+\sum_{k=1}^{2}\sum_{j\in N_i}\left(\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\right)^2\right)-\frac{\partial\alpha_{i,1}}{\partial\hat{\boldsymbol{W}}_{i,1}}\dot{\hat{\boldsymbol{W}}}_{i,1}-\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\left(x_{j,k+1}+f_{j,k}\right)-\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial y_0^{(k)}}y_0^{(k+1)}-\frac{\partial\alpha_{i,1}}{\partial r_i}\dot{r}_i-\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial\psi^{(k)}}\psi^{(k+1)}+\frac{z_{i,2}}{2\left(k_{b2}^2-z_{i,2}^2\right)}+\frac{\left(k_{b2}^2-z_{i,2}^2\right)}{z_{i,2}}\tanh^2\left(\frac{z_{i,2}}{\delta_{i,2}}\right)\sum_{k=1}^{2}\sum_{l=1}^{k}\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right).$

According to Lemma 6, we obtain

$$\dot{\Xi}_{i,2}\le-\frac{k_{i,1}z_{i,1}^2}{k_{b1}^2-z_{i,1}^2}+\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\left(z_{i,3}+\alpha_{i,2}+\boldsymbol{W}_{i,2}^T\boldsymbol{P}_{m_{i,2}}+E_{i,2}\right)$$

$$-\frac{z_{i,2}^2\dot{k}_{b2}}{k_{b2}\left(k_{b2}^2-z_{i,2}^2\right)}-\frac{1}{2}\frac{z_{i,2}^2}{\left(k_{b2}^2-z_{i,2}^2\right)^2}$$
$$+\gamma_{i,1}\tilde{\boldsymbol{W}}_{i,1}^T\dot{\hat{\boldsymbol{W}}}_{i,1}-\tilde{\boldsymbol{W}}_{i,2}^T\dot{\hat{\boldsymbol{W}}}_{i,2}$$
$$+\frac{1}{2}\sum_{q=1}^{2}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}$$
$$\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$$
$$+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\varepsilon_{i,1}^2+\frac{1}{2}\ell_{i,1}^2+\frac{1}{2}\ell_{i,1}^2$$
$$+\frac{1}{2}\ell_{i,2}^2+\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\ell_{j,k}^2 \tag{32}$$

Accordingly, the virtual controller $\alpha_{i,2}$ and adaptive law $\dot{\hat{\boldsymbol{W}}}_{i,2}$ are designed as follows

$$\alpha_{i,2}=-\hat{\boldsymbol{W}}_{i,2}^T\boldsymbol{P}_{m_{i,2}}-k_{i,2}z_{i,2}-\bar{\eta}_{i,2}z_{i,2} \tag{33}$$
$$\dot{\hat{\boldsymbol{W}}}_{i,2}=\frac{z_{i,2}}{\left(k_{b2}^2-z_{i,2}^2\right)}\boldsymbol{P}_{m_{i,2}}-\gamma_{i,2}\hat{\boldsymbol{W}}_{i,2} \tag{34}$$

where constants $k_{i,2}>0$, $\gamma_{i,2}>0$ and $\bar{\eta}_{i,2}=\sqrt{\left(\frac{\dot{k}_{b2}}{k_{b2}}\right)^2+\eta_{i,2}}$. Obviously, $\bar{\eta}_{i,2}+\frac{\dot{k}_{b2}}{k_{b2}}\geq 0$, therefore, $-\frac{z_{i,2}^2}{k_{b2}^2-z_{i,2}^2}\left(\bar{\eta}_{i,2}+\frac{\dot{k}_{b2}}{k_{b2}}\right)\leq 0$.

Substituting (33) and (34) into (32), we have

$$\dot{\Xi}_{i,2}\leq-\sum_{l=1}^{2}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{z_{i,2}z_{i,3}}{k_{b2}^2-z_{i,2}^2}+\sum_{l=1}^{2}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}$$
$$+\frac{1}{2}\sum_{q=1}^{2}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}$$
$$\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$$
$$+\frac{1}{2}\sum_{l=1}^{2}\varepsilon_{i,l}^2+\frac{1}{2}\ell_{i,1}^2+\frac{1}{2}\sum_{k=1}^{2}\ell_{i,k}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2$$
$$+\frac{1}{2}\sum_{k=1}^{2}\sum_{j\in N_i}\ell_{j,k}^2 \tag{35}$$

Step p $(3\leq p\leq n-1)$: See Appendix.

Step n: Design the candidate Lyapunov function $\Xi_{i,n}$ as follows

$$\Xi_{i,n}=\Xi_{i,n-1}+\frac{1}{2}\ln\frac{k_{bn}^2}{k_{bn}^2-z_{i,n}^2}+\frac{1}{2}\tilde{\boldsymbol{W}}_{i,n}^T\tilde{\boldsymbol{W}}_{i,n}$$
$$+\frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{k}\int_{t-\tau_{i,k}}^{t}\varphi_{i,k,l}^2\left(x_{i,l}(s)\right)ds$$
$$+\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{n}\sum_{l=1}^{k}\int_{t-\tau_{j,k}}^{t}\varphi_{j,k,l}^2\left(x_{j,l}(s)\right)ds \tag{36}$$

where $\tilde{\boldsymbol{W}}_{i,n}=\boldsymbol{W}_{i,n}-\hat{\boldsymbol{W}}_{i,n}$ is estimation error, $\hat{\boldsymbol{W}}_{i,n}$ is the estimate of $\boldsymbol{W}_{i,n}$ and $k_{bn}=\kappa_{i,n}-\iota_{i,n}$ with $\iota_{i,n}$ is a constant.

Calculating the derivative of $\Xi_{i,n}$, and taking (1) and (5) into account, we have

$$\dot{\Xi}_{i,n}=\dot{\Xi}_{i,n-1}-\tilde{\boldsymbol{W}}_{i,n}^T\dot{\hat{\boldsymbol{W}}}_{i,n}$$
$$+\frac{z_{i,n}}{\left(k_{bn}^2-z_{i,n}^2\right)}$$
$$\left(u_i+f_{i,n}+\phi_{i,n}\left(\bar{\boldsymbol{x}}_{i,n}\left(t_{i,n}^{\tau}\right)\right)\right.$$
$$\left.+\Delta_{i,n}-\dot{\alpha}_{i,n-1}-\frac{z_{i,n}\dot{k}_{bn}}{k_{bn}}\right)$$
$$+\frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{k}\left[\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)-\varphi_{i,k,l}^2\left(x_{i,l}\left(t_{i,k}^{\tau}\right)\right)\right]$$
$$+\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{n}\sum_{l=1}^{k}$$
$$\left[\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)-\varphi_{j,k,l}^2\left(x_{j,l}\left(t_{j,k}^{\tau}\right)\right)\right] \tag{37}$$

where $\dot{\alpha}_{i,n-1}=\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial x_{i,k}}\left(x_{i,k+1}+f_{i,k}\right.$ $\left.+\phi_{i,k}\left(\bar{\boldsymbol{x}}_{i,k}\left(t_{i,k}^{\tau}\right)\right)+\Delta_{i,k}\right)+\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\boldsymbol{W}}_{i,k}}\dot{\hat{\boldsymbol{W}}}_{i,k}$ $+\sum_{k=0}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial y_0^{(k)}}y_0^{(k+1)}+\sum_{k=1}^{n}\sum_{j\in N_i}\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}\left(x_{j,k+1}+f_{j,k}\right.$ $\left.+\phi_{j,k}\left(\bar{\boldsymbol{x}}_{j,k}\left(t_{j,k}^{\tau}\right)\right)+\Delta_{j,k}\right)$.

Substituting (67) into (37), and using Lemma 5 along with Assumption 3 and Assumption 4, we have

$$\dot{\Xi}_{i,n}=-\sum_{l=1}^{n-1}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{z_{i,n}}{\left(k_{bn}^2-z_{i,n}^2\right)}\left(u_i+F_{i,n}\right)$$

$$- \frac{z_{i,n}^2 \dot{k}_{bn}}{k_{bn}\left(k_{bn}^2 - z_{i,n}^2\right)} + \sum_{l=1}^{n-1} \gamma_{i,l} \tilde{\boldsymbol{W}}_{i,l}^T \hat{\boldsymbol{W}}_{i,l}$$

$$- \tilde{\boldsymbol{W}}_{i,n}^T \dot{\hat{\boldsymbol{W}}}_{i,n} - \frac{z_{i,n}^2}{2\left(k_{bn}^2 - z_{i,n}^2\right)}$$

$$+ \frac{1}{2}\sum_{q=1}^{n-1}\left(1 - 2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}$$

$$\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$$

$$+ \frac{1}{2}\sum_{l=1}^{n-1}\varepsilon_{i,l}^2 + \frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2 + \frac{1}{2}\sum_{j\in N_i} a_{ij}\ell_{j,1}^2$$

$$+ \frac{1}{2}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \tag{38}$$

where $F_{i,n} = f_{i,n} - \sum_{k=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{i,k}}\left(x_{i,k+1} + f_{i,k}\right) - \sum_{k=1}^{n}\sum_{j\in N_i}\frac{\partial \alpha_{i,n-1}}{\partial x_{j,k}}\left(x_{j,k+1} + f_{j,k}\right) + \frac{z_{i,n}}{\left(k_{bn}^2 - z_{i,n}^2\right)}\left(1 + \sum_{k=1}^{n-1}\left(\frac{\partial \alpha_{i,n-1}}{\partial x_{i,k}}\right)^2 + \sum_{k=1}^{n}\sum_{j\in N_i}\left(\frac{\partial \alpha_{i,n-1}}{\partial x_{j,k}}\right)^2\right) - \sum_{k=1}^{n-1}\frac{\partial \alpha_{i,n-1}}{\partial \hat{\boldsymbol{W}}_{i,k}}\dot{\hat{\boldsymbol{W}}}_{i,k} + \frac{z_{i,n}}{2\left(k_{bn}^2 - z_{i,n}^2\right)} - \sum_{k=0}^{n-1}\frac{\partial \alpha_{i,n-1}}{\partial y_0^{(k)}} y_0^{(k+1)} + \frac{k_{bn}^2 - z_{i,n}^2}{z_{i,n}} \tanh^2\left(\frac{z_{i,n}}{\delta_{i,n}}\right)\sum_{k=1}^{n}\sum_{l=1}^{k}\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$.

According to Lemma 6, we obtain

$$\dot{\Xi}_{i,n} = -\sum_{l=1}^{n-1}\frac{k_{i,l} z_{i,l}^2}{k_{bl}^2 - z_{i,l}^2} + \frac{z_{i,n}}{\left(k_{bn}^2 - z_{i,n}^2\right)}$$

$$\left(u_i + \boldsymbol{W}_{i,n}^T \boldsymbol{P}_{m_{i,n}} + E_{i,n}\right)$$

$$- \frac{z_{i,n}^2 \dot{k}_{bn}}{k_{bn}\left(k_{bn}^2 - z_{i,n}^2\right)} - \frac{z_{i,n}^2}{2\left(k_{bn}^2 - z_{i,n}^2\right)}$$

$$+ \sum_{l=1}^{n-1}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T \hat{\boldsymbol{W}}_{i,l} - \tilde{\boldsymbol{W}}_{i,n}^T \dot{\hat{\boldsymbol{W}}}_{i,n}$$

$$+ \frac{1}{2}\sum_{q=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}$$

$$\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$$

$$+ \frac{1}{2}\sum_{l=1}^{n-1}\varepsilon_{i,l}^2 + \frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2 + \frac{1}{2}\sum_{j\in N_i} a_{ij}\ell_{j,1}^2$$

$$+ \frac{1}{2}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \tag{39}$$

In addition, we have

$$\frac{z_{i,n}}{k_{bn}^2 - z_{i,n}^2}E_{i,n} \le \frac{1}{2}\frac{z_{i,n}^2}{\left(k_{bn}^2 - z_{i,n}^2\right)^2} + \frac{1}{2}\varepsilon_{i,n}^2 \tag{40}$$

where $\varepsilon_{i,n} > 0$ is a constant.

By substituting (40) into (39), the following result is obtained

$$\dot{\Xi}_{i,n} = -\sum_{l=1}^{n-1}\frac{k_{i,l} z_{i,l}^2}{k_{bl}^2 - z_{i,l}^2} + \frac{z_{i,n}}{\left(k_{bn}^2 - z_{i,n}^2\right)}\left(u_i + \boldsymbol{W}_{i,n}^T \boldsymbol{P}_{m_{i,n}}\right)$$

$$- \frac{z_{i,n}^2 \dot{k}_{bn}}{k_{bn}\left(k_{bn}^2 - z_{i,n}^2\right)}$$

$$+ \frac{1}{2}\sum_{q=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}$$

$$\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)$$

$$- \tilde{\boldsymbol{W}}_{i,n}^T \dot{\hat{\boldsymbol{W}}}_{i,n} + \sum_{l=1}^{n-1}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T \hat{\boldsymbol{W}}_{i,l} + \frac{1}{2}\sum_{l=1}^{n}\varepsilon_{i,l}^2$$

$$+ \frac{1}{2}\sum_{j\in N_i} a_{ij}\ell_{j,1}^2 + \frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2$$

$$+ \frac{1}{2}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \tag{41}$$

The following ETC scheme is developed with the goal of minimizing controller update frequency and reducing both communication burden and energy consumption in MASs.

$$u_i(t) = \sigma_i\left(t_{i,k}\right) \quad \forall t \in \left[t_{i,k}, t_{i,k+1}\right) \tag{42}$$

$$t_{i,k+1} = \inf\left\{t \in R \,\middle|\, |e_i(t)| \ge \lambda_i |u_i(t)| + \chi_i\right\} \tag{43}$$

where, $\inf\{\cdot\}$ denotes the infimum, measurement errors $e_i(t) = \sigma_i(t) - u_i(t)$, $0 < \lambda_i < 1$, $\chi_i > 0$, $\chi_i' > \frac{\chi_i}{1-\lambda_i}$

are design parameters, $t_{i,k}\left(k\in Z^{+}\right)$ denotes the kth triggering instant of the ith agent. The ETC strategy specifies that from triggering instant $t_{i,k}$ to the next triggering instant $t_{i,k+1}$, Between the current and the next triggering instants, the control input $u_i\ (t)$ is maintained at the value σ_i, and is revised upon the arrival of the subsequent triggering time $t_{i,k+1}$.

Remark 4 It should be noted that the tuning of the triggering parameters λ_i and χ_i must balance communication efficiency and tracking performance. Parameter λ_i regulates the trade-off between event-triggering frequency and steady-state error, while χ_i determines the minimum inter-event time and must satisfy $\chi_i' > \frac{\chi_i}{1-\lambda_i}$ to ensure stability. Therefore, during parameter tuning, it is advisable to first select a moderate value of λ_i (e.g., within the range of 0.2 to 0.8) to balance performance and communication cost. Subsequently, based on the allowable error tolerance, set χ_i slightly below the predefined threshold $(1-\lambda_i)\,\chi_i'$, and then gradually decrease χ_i to reduce steady-state error while monitoring the triggering intervals. Finally, iteratively adjust both parameters to achieve the desired trade-off between control update frequency and tracking accuracy.

The adaptive control law and adaptive update law are designed as follows

$$\sigma_i(t) = -(1+\lambda_i)\left(\alpha_{i,n}\tanh\left(\frac{z_{i,n}\alpha_{i,n}}{(k_{bn}^2-z_{i,n}^2)\varsigma_i}\right) + \chi_i'\tanh\left(\frac{z_{i,n}\chi_i'}{(k_{bn}^2-z_{i,n}^2)\varsigma_i}\right)\right) \tag{44}$$

$$\alpha_{i,n} = -\hat{\boldsymbol{W}}_{i,n}^T\boldsymbol{P}_{m_{i,n}} - k_{i,n}z_{i,n} - \bar{\eta}_{i,n}z_{i,n} \tag{45}$$

$$\dot{\hat{\boldsymbol{W}}}_{i,n} = \frac{z_{i,n}}{k_{bn}^2-z_{i,n}^2}\boldsymbol{P}_{m_{i,n}} - \gamma_{i,n}\hat{\boldsymbol{W}}_{i,n} \tag{46}$$

where constants $k_{i,n} > 0$ and $\gamma_{i,n} > 0$, $\bar{\eta}_{i,n} = \sqrt{\left(\frac{\dot{k}_{bn}}{k_{bn}}\right)^2 + \eta_{i,n}}$. Obviously, $\bar{\eta}_{i,n} + \frac{\dot{k}_{bn}}{k_{bn}} \geq 0$, therefore, $-\frac{z_{i,n}^2}{k_{bn}^2-z_{i,n}^2}\left(\bar{\eta}_{i,n} + \frac{\dot{k}_{bn}}{k_{bn}}\right) \leq 0$.

From (43), we have

$$\sigma_i\ (t) = (1+\omega_1\ (t)\ \lambda_i)\ u_i\ (t) + \omega_2\ (t)\ \chi_i \tag{47}$$

where $t \in \left[t_{i,k}, t_{i,k+1}\right]$, $\omega_1\ (t)$ and $\omega_2\ (t)$ satisfy $|\omega_1\ (t)| \leq 1$ and $|\omega_2\ (t)| \leq 1$.

By manipulating (47), we obtain

$$u_i\ (t) = \frac{\sigma_i\ (t)}{1+\omega_1\ (t)\ \lambda_i} - \frac{\omega_2\ (t)\ \chi_i}{1+\omega_1\ (t)\ \lambda_i} \tag{48}$$

Since $\forall\varsigma > 0$ and $\xi \in R$, $-\xi\tanh\left(\frac{\xi}{\varsigma}\right) \leq 0$, from (43), we obtain $z_{i,n}\sigma_i\ (t) \leq 0$, Furthermore, with the additional constraint $|\omega_1\ (t)| \leq 1$, $|\omega_2\ (t)| \leq 1$, we consequently obtain

$$\frac{z_{i,n}\sigma_i\ (t)}{\left(k_{bn}^2-z_{i,n}^2\right)(1+\omega_1\ (t)\ \lambda_i)} \leq \frac{z_{i,n}\sigma_i\ (t)}{\left(k_{bn}^2-z_{i,n}^2\right)(1+\lambda_i)} \tag{49}$$

$$\left|\frac{\omega_2\ (t)\ \chi_i}{1+\omega_1\ (t)\ \lambda_i}\right| \leq \frac{\chi_i}{1-\lambda_i} \tag{50}$$

It follows from (48), (49) and (50) that

$$\frac{z_{i,n}}{k_{bn}^2-z_{i,n}^2}u_i(t) \leq 0.557\varsigma_i + \left|\frac{z_{i,n}\chi_i}{(k_{bn}^2-z_{i,n}^2)(1-\lambda_i)}\right| - \left|\frac{z_{i,n}\alpha_{i,n}}{k_{bn}^2-z_{i,n}^2}\right| - \left|\frac{z_{i,n}\chi_i'}{k_{bn}^2-z_{i,n}^2}\right|$$

From the above analysis, it follows that

$$\begin{aligned}\dot{\Xi}_{i,n} = &-\sum_{l=1}^{n-1}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2} + \frac{z_{i,n}\boldsymbol{W}_{i,n}^T\boldsymbol{P}_{m_{i,n}}}{k_{bn}^2-z_{i,n}^2} - \left|\frac{z_{i,n}\alpha_{i,n}}{k_{bn}^2-z_{i,n}^2}\right| \\ &- \left|\frac{z_{i,n}\chi_i'}{k_{bn}^2-z_{i,n}^2}\right| \\ &+ \left|\frac{z_{i,n}\chi_i}{(k_{bn}^2-z_{i,n}^2)(1-\lambda_i)}\right| - \frac{z_{i,n}^2\dot{k}_{bn}}{k_{bn}(k_{bn}^2-z_{i,n}^2)} \\ &+ \sum_{l=1}^{n-1}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l} - \tilde{\boldsymbol{W}}_{i,n}^T\dot{\hat{\boldsymbol{W}}}_{i,n} \\ &+ \frac{1}{2}\sum_{q=1}^{n}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k} \\ &\left(\varphi_{i,k,l}^2(x_{i,l}(t)) + \sum_{j\in N_i}\varphi_{j,k,l}^2(x_{j,l}(t))\right) \\ &+ 0.557\varsigma_i + \frac{1}{2}\sum_{l=1}^{n}\varepsilon_{i,l}^2 + \frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2 \\ &+ \frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2 + \frac{1}{2}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2\end{aligned} \tag{51}$$

Substituting (45) and (46) into (51) gains

$$\begin{aligned}\dot{\Xi}_{i,n} =& -\sum_{l=1}^{n}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\sum_{l=1}^{n}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}\\ &+\frac{1}{2}\sum_{q=1}^{n}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}\\ &\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)\\ &+0.557\varsigma_i+\frac{1}{2}\sum_{l=1}^{n}\varepsilon_{i,l}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2\\ &+\frac{1}{2}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2\\ &+\frac{1}{2}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \end{aligned} \tag{52}$$

4 Stability analysis

Based on the preceding discussion, the stability of the closed-loop system is investigated based on Lyapunov theory, and the primary findings are summarized in the subsequent theorem.

Theorem 1 *Consider the MASs (1) quipped with the adaptive laws (21), (34), (66), (46), the virtual controllers (20), (33), (65), and the actual controller (45). Under the action of the actual controller and the event-triggered mechanism (42), (43), the following properties hold*

i) The MAS is semi-globally bounded, and the consensus output error of all agents satisfies predetermined performance specifications.
ii) All system states are ensured to evolve within the specified constraint bounds.
iii) The minimal inter-event interval is strictly positive, thereby excluding Zeno behavior.

Proof For the closed-loop systems, the Lyapunov function candidate Ξ is considered as follows

$$\Xi=\sum_{i=1}^{N}\Xi_{i,n} \tag{53}$$

By taking the derivative of both sides of equation (53), we obtain

$$\begin{aligned}\dot{\Xi} \le& -\sum_{i=1}^{N}\sum_{l=1}^{n}\left(\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^2}{2}\right)+\sum_{i=1}^{N}0.557\varsigma_i\\ &+\frac{1}{2}\sum_{i=1}^{N}\sum_{l=1}^{n}\varepsilon_{i,l}^2\\ &+\frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}\\ &\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)\\ &+\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2\\ &+\frac{1}{2}\sum_{i=1}^{N}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \end{aligned} \tag{54}$$

where $\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}\le-\frac{\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^2}{2}+\frac{\gamma_{i,l}\boldsymbol{W}_{i,l}^2}{2}$.

Thus, we have

$$\dot{\Xi}\le-a\Xi+C \tag{55}$$

where $C=\frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right)\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right)+\sum_{i=1}^{N}0.557\varsigma_i+\frac{1}{2}\sum_{i=1}^{N}\sum_{l=1}^{n}\varepsilon_{i,l}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2$, and $a=\min\left\{\frac{k_{i,l}}{2},\gamma_{i,l}\right\}$.

Multiplying both sides of inequality (55) by e^{at} and integrating over the interval $[0,t]$, we obtain

$$\int_0^t e^{at}\dot{\Xi}dt\le\int_0^t -ae^{at}\Xi dt+\int_0^t e^{at}Cdt$$

Thus,

$$\frac{1}{2}z_{i,1}^2\le\Xi(t)\le\left(\Xi(0)-\frac{C}{a}\right)e^{-at}+\frac{C}{a} \tag{56}$$

Hence $z_{i,1} \leq \underline{k}_{b1}$, where $\underline{k}_{b1} = \sqrt{2\left(\Xi(0) - \frac{C}{a}\right)e^{-at} + \frac{2C}{a}}$.

Applying Lemma 1, due to $\left|z_{i,1}\right| \leq \underline{k}_{b1} \leq k_{b1}$, we conclude that

$$\|Y - Y_d\| \leq \frac{\underline{k}_{b1}}{\sigma_{\min}} < \frac{k_{b1}}{\sigma_{\min}} \tag{57}$$

From (57), it can be concluded that by appropriately selecting design parameters, the tracking error converges to a bounded region centered at the origin. Therefore, every agent achieves tracking of the expected trajectory.

Combining (6) with (56), we derive

$$\left|z_{i,l}\right| \leq k_{bl}\sqrt{1 - e^{-2(\Xi(0)-C/a)e^{-at+2C/a}}} \leq k_{bl} \tag{58}$$

Because of $|y_d| < \Gamma_1$, we can obtain $|y_1| < \frac{k_{b1}}{\sigma_{\min}} + \Gamma_1$. Define $k_{b1} = \sigma_{\min}(\kappa_1 - \Gamma_1)$ and, therefore, $|y_1| = \left|x_{i,1}\right| < \kappa_1$. According to (20), $\alpha_{i,1}$ consists of $\tilde{\boldsymbol{W}}_{i,1}$, $z_{i,1}$ and $\dot{y}_d$, where both $z_{i,1}$ and $\tilde{\boldsymbol{W}}_{i,1}$ are bounded. Moreover, $\boldsymbol{W}_{i,1}$ as the true value of $\tilde{\boldsymbol{W}}_{i,1}$, is bounded. Based on $\tilde{\boldsymbol{W}}_{i,1} = \boldsymbol{W}_{i,1} - \hat{\boldsymbol{W}}_{i,1}$, $\hat{\boldsymbol{W}}_{i,1}$ is also bounded, it can be concluded that $\alpha_{i,1}$ is bounded as well. Without loss of generality, we assume $\alpha_{i,1} < \bar{\alpha}_{i,1}$. Since $z_{i,2} = x_{i,2} - \alpha_{i,1}$ holds, it follows from (58) that $\left|z_{i,2}\right| \leq k_{b2}$, Define $k_{b2} = \kappa_2 - \bar{\alpha}_{i,1}$, hence, $\left|x_{i,2}\right| < \kappa_2$. Similarly, it can be proved that $\left|x_{i,l}\right| < \kappa_l$, where $i = 1, \ldots, N$, $l = 3, 4, \ldots, n$.

To conclude, the adaptive ETC scheme is proven to guarantee that all system states remain within the predefined constraints. Zeno behavior refers to the phenomenon where infinitely many event triggers occur within a finite time interval. To reduce the number of control actuation updates and conserve communication resources, it is essential to prevent the emergence of Zeno behavior.

From $e_i(t) = \sigma_i(t) - u_i(t), \forall t \in \left[t_{i,k}, t_{i,k+1}\right)$, it follows that $\frac{d}{dt}|e_i| = \frac{d}{dt}(e_i \times e_i)^{\frac{1}{2}} = sign(e_i)\dot{e}_i \leq |\dot{\sigma}_i|$, where $\dot{\sigma}_i$ denotes the derivative of σ_i. Since all signals within the system are guaranteed to remain bounded, there necessarily exists a positive constant μ, such that $|\dot{\sigma}_i| \leq \mu$. Given $e_i(t_k) = 0$ and $\lim_{t \to t_{k+1}} e_i(t) = \chi_i$, it follows that the lower bound t^* of the inter-event interval satisfies $t^* \geq \frac{\chi_i}{\mu}$, This conclusively demonstrates that the protocol proposed in this work excludes Zeno behavior.

Three separate cases are considered for analyzing the consensus error of the MAS.

Case 1: $z_{i,l} \in \Omega_{z_{i,l}}$ for $i = 1, 2, ..., N, l = 1, 2, ..., n$, then $\left|z_{i,l}\right| < 0.8814\delta_{i,l}$. The consensus tracking error e_1 is derived as $e_1 = (L + \mathrm{B})(Y - Y_d)$, in accordance with definition $z_{i,1}$. Consequently, we have $\|Y - Y_d\| \leq \frac{\|e_1\|}{\sigma_{\min}}$ with $\sigma_{\min}$ is the smallest singular value of matrix $L + \mathrm{B}$. Therefore, the tracking error $Y - Y_d$ is bounded. Moreover, by selecting the design parameter $\delta_{i,1}$ to be sufficiently small, the consensus error can be reduced to an arbitrarily low level.

Case 2: $z_{i,l} \notin \Omega_{z_{i,l}}$, then $1 - 2\tanh^2\left(\frac{z_{i,l}}{\delta_{i,l}}\right) \leq 0$. Thus, we have

$$\frac{1}{2}\left(1 - 2\tanh^2\left(\frac{z_{i,l}}{\delta_{i,l}}\right)\right)\sum_{k=1}^{l}\sum_{l=1}^{k}\left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j \in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right) \leq 0$$

and $\dot{V} < 0$, if $\sum_{i=1}^{N}\sum_{l=1}^{n}\left(\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2 - z_{i,l}^2} + \frac{\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^2}{2}\right) > B$, where $B = \sum_{i=1}^{N} 0.557\varsigma_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{l=1}^{n}\varepsilon_{i,l}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j \in N_i} a_{ij}\ell_{j,1}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n}\sum_{k=1}^{q}\ell_{i,k}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{q=2}^{n}\sum_{k=1}^{q}\sum_{j \in N_i}\ell_{j,k}^2$, then it follows that $\|e_1\| \leq \sqrt{\frac{B}{k}}$, where $k = \min\left\{k_{i,l}, i = 1, 2, ..., N, l = 1, 2, ..., n\right\}$, therefore, the tracking error $Y - Y_d$ is bounded. Moreover, by selecting the design parameter $\delta_{i,1}$ to be sufficiently small, the consensus error can be reduced to an arbitrarily low level.

Case 3: Under this scenario, the set $z_{i,l}$ is partitioned into two disjoint subsets $z_{i_1,l_1} \in \Omega_{i_1,l_1}$ and $z_{i_2,l_2} \notin \Omega_{i_2,l_2}$, where $i_1 \in S_{i_1}, l_1 \in S_{l_1}, i_2 \in S_{i_2}, l_2 \in S_{l_2}$, Based on Case 1, if $z_{i_1,l_1} \in \Omega_{z_{i_1,l_1}}$, then $\left|z_{i_1,l_1}\right| < 0.8814\delta_{i_1,l_1}$, Therefore, $z_{i_1,1}$ is uniformly bounded over $\forall i_1 \in S_{i_1}$. Based on Case 2, when $z_{i_2,l_2} \notin \Omega_{i_2,l_2}$, we have $\frac{1}{2}\left(1 - 2\tanh^2\left(\frac{z_{i_2,l_2}}{\delta_{i_2,l_2}}\right)\right)\sum_{k \in S_{l_2}}\sum_{l=1}^{k} \left(\varphi_{i,k,l}^2\left(x_{i,l}(t)\right) + \sum_{j \in N_i}\varphi_{j,k,l}^2\left(x_{j,l}(t)\right)\right) \leq 0$. Consequently, $z_{i_2,1}$ is bounded for $\forall i_2 \in S_{i_2}$. □

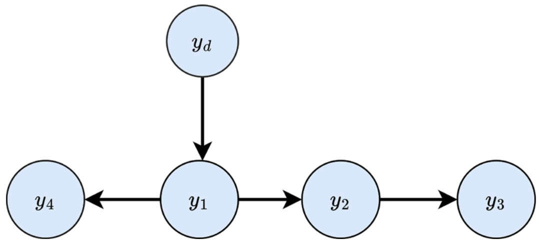
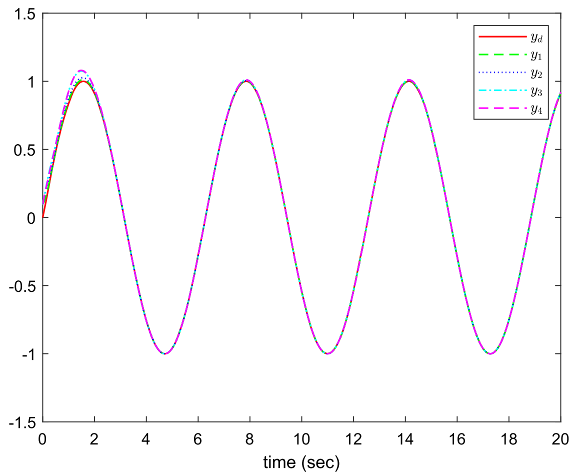
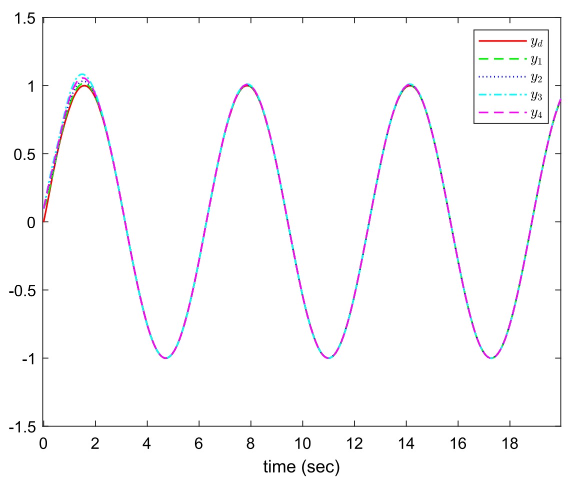


Fig. 2 Communication topology



(a) System output y_i and reference signal y_d in Case 1.

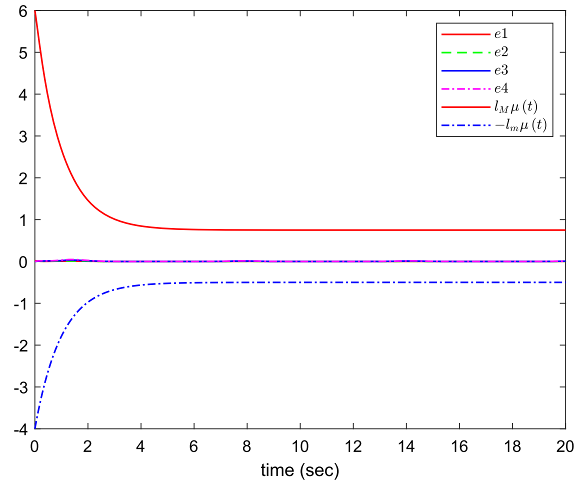


(b) System output y_i and reference signal y_d in Case 2.

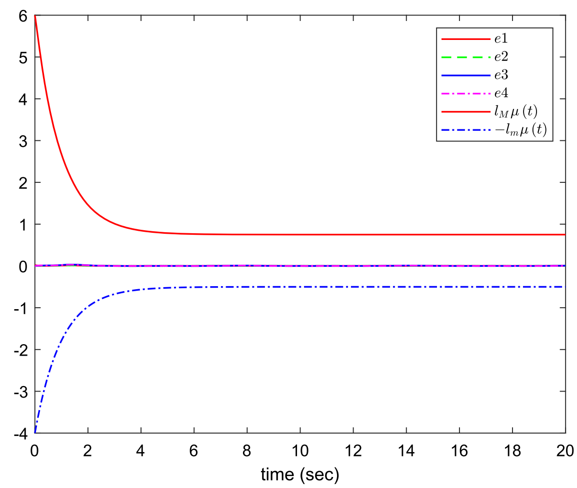
Fig. 3 System output y_i and reference signal y_d in Example 1

5 Simulation example

This section presents three simulation examples to demonstrate the effectiveness of the developed control approach. A nonlinear MAS consisting of a virtual leader and four followers is considered, with its communication topology illustrated in Fig. 2.



(a) Tracking error e_i in Case 1.

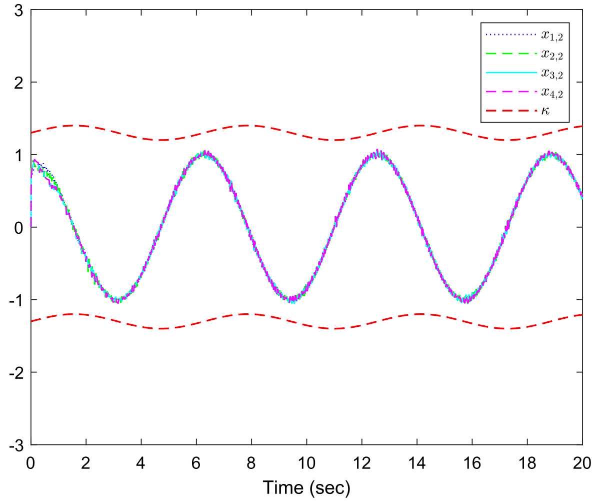


(b) Tracking error e_i in Case 2.

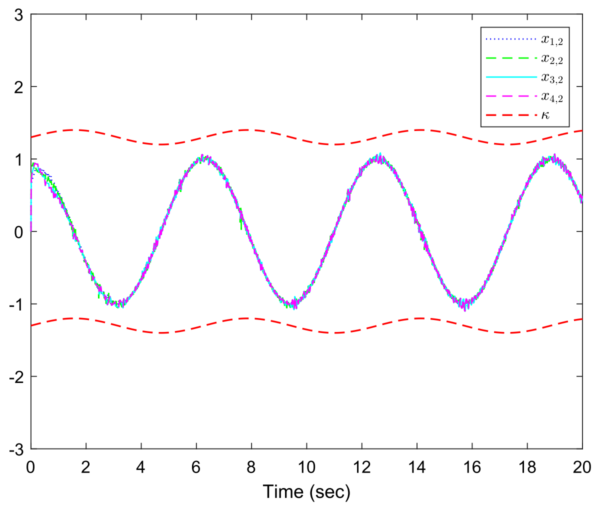
Fig. 4 Tracking error e_i in Example 1

Accordingly, the Laplacian matrix is derived as $L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$.

Fig. 5 System state $x_{i,2}$ in Example 1



(a) System state $x_{i,2}$ in Case 1.



(b) System state $x_{i,2}$ in Case 2.

Example 1 Consider the nonlinear MAS model with state time-delays as follow

$$\begin{cases} \dot{x}_{\mathrm{i}1} = x_{\mathrm{i}2} + f_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\right) + \phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\left(t-\tau_{i,1}\right)\right) \\ \dot{x}_{\mathrm{i}2} = u_i + f_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\right) + \phi_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\left(t-\tau_{i,2}\right)\right) \\ +\Delta_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\left(t\right)\right) \\ y_i = x_{\mathrm{i}1} \end{cases} \tag{59}$$

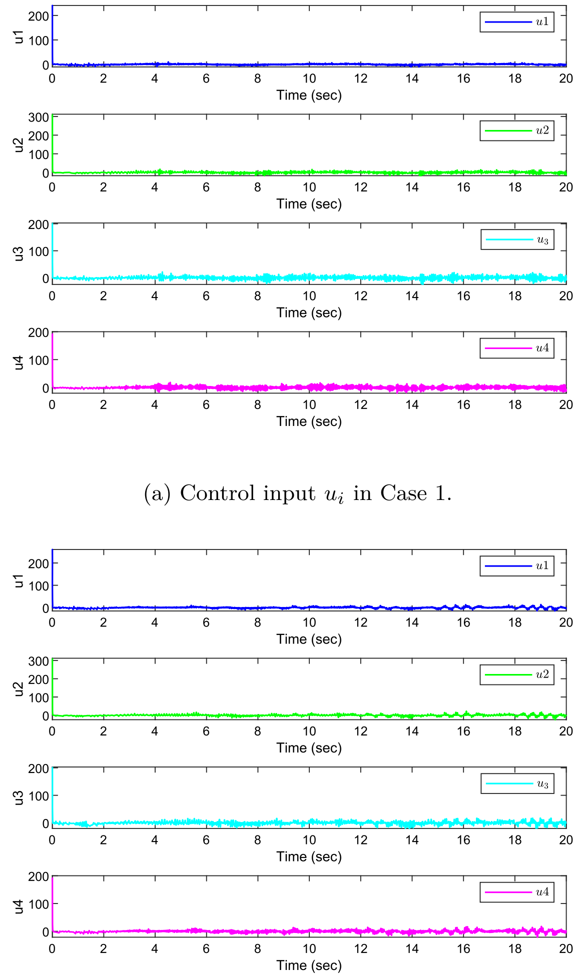
where $f_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\right) = 0.1x_{\mathrm{i},1}e^{x_{i,1}}$, $f_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\right) = x_{\mathrm{i},1}x_{i,2}^2$, $\phi_{i,1}\left(\bar{\boldsymbol{x}}_{i,1}\right) = 0.05\sin\left(x_{i,1}\right)$, $\phi_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\right) = 0.05\sin\left(x_{i,1}x_{i,2}\right)$, $\Delta_{i,2}\left(\bar{\boldsymbol{x}}_{i,2}\left(t\right)\right) = 0.1x_{\mathrm{i},1}x_{i,2}\cos(t)$. $\bar{\boldsymbol{x}}_{1,2}\left(0\right) = [0,0]^T$, $\bar{\boldsymbol{x}}_{2,2}\left(0\right) = [0,0]^T$, $\bar{\boldsymbol{x}}_{3,2}\left(0\right) = [0,0]^T$, $\bar{\boldsymbol{x}}_{4,2}\left(0\right) = [0,0]^T$. The virtual leader's reference signal is set as follow $y_d = \sin\left(t\right)$.

The parameters for the prescribed performance and state constraints are assigned as follows respectively: $\psi_0 = 4$, $\psi_\infty = 0.5$, $\upsilon = 1$, $l_m = 1$, $l_M = 1.5$, $\kappa_1 = 1.3 + 0.1\sin\left(t\right)$, $\kappa_2 = -1.3 + 0.1\sin\left(t\right)$. The value of the event-triggered parameter is selected as: $\eta_{i,1} = 5$, $\eta_{i,2} = 2$, $\lambda_i = 0.5$, $\chi_i = 1$, $\chi_1' = 45$, $\chi_2' = 25$, $\chi_3' = 35$, $\chi_4' = 30$, $\varsigma_1 = \varsigma_4 = 0.4$, $\varsigma_2 = 0.2$, $\varsigma_3 = 0.8$, $M_1 = 0.5$, $M_2 = 0.4$, $M_3 = 0.6$, $M_4 = 0.3$. To further illustrate the stability of the model, two sets of different time delays and controller design parameters were selected as follows:

(i) Case 1: The time delay is given by $\tau_{i,1} = \tau_{i,2} = 0.3s$. The following values are assigned to the controller parameters: $k_{1,1} = 35$, $k_{2,1} = k_{4,1} = 55$, $k_{3,1} = 25$, $\gamma_{1,1} = \gamma_{3,1} = \gamma_{4,1} = 1000$, $\gamma_{2,1} = 5000$, $k_{b1} = 5$, $k_{b2} = 4$, $\gamma_{12} = 0.1$, $\gamma_{2,2} = \gamma_{3,2} = \gamma_{4,2} = 1$, $k_{1,2} = 200$, $k_{2,2} = 500$, $k_{3,2} = k_{4,2} = 100$.

(ii) Case 2: The time delay is given by $\tau_{i,1} = \tau_{i,2} = 0.6s$. The following values are assigned to the controller parameters: $k_{1,1} = 75$, $k_{2,1} = 95$, $k_{3,1} = 30$, $k_{4,1} = 35$, $\gamma_{1,1} = \gamma_{4,1} = 500$, $\gamma_{2,1} = \gamma_{3,1} = 1000$, $k_{b1} = 5$, $k_{b2} = 4$, $\gamma_{12} = 0.5$, $\gamma_{2,2} = \gamma_{3,2} = \gamma_{4,2} = 10$, $k_{1,2} = k_{2,2} = 200$, $k_{3,2} = k_{4,2} = 100$.

The simulation results are presented in Fig. 3, 4, 5, 6,7 and Tables 1 and 2. Fig. 3 shows the tracking performance of the four follower agents y_i $(i = 1, 2, 3, 4)$ with respect to the virtual leader's reference signal y_d. It clearly demonstrates that the outputs of the agents can effectively track the reference signal. Fig. 4 presents the tracking error, which verifies that the output errors of the system satisfy the prescribed performance conditions. From the state trajectories in Fig. 5, it can be observed that all the agent states remain within the predefined constraint bounds. Fig. 6 displays the control inputs u_i of the four agents. Fig. 7 depicts the time intervals between two successive triggering events for each agent. Combined with Table 1, it can be seen that the four follower agents have different numbers of triggers, which effectively reduces the frequency of controller updates and further confirms that Zeno behavior does not occur. Table 2 compares the triggering law of the ETC scheme proposed in this paper with that of the fixed ETC scheme, thereby verifying the effectiveness of the proposed ETC strategy.



(a) Control input u_i in Case 1.

(b) Control input u_i in Case 2.

Fig. 6 Control input u_i in Example 1

Example 2 To further validate the effectiveness of the proposed scheme, a practical inverted pendulum system [51] is considered.

The dynamic model of the inverted pendulum system is formulated as follows

$$\begin{cases} \dot{\varpi}_{i1} = \varpi_{i2} + f_{i,1}\left(\varpi_{i,1}\right) + h_{i,1} \\ \dot{\varpi}_{i2} = \frac{u_i}{L_i} + f_{i,2}\left(\bar{\boldsymbol{\varpi}}_{i,2}\right) + h_{i,2} \\ y_i = \varpi_{i1} \end{cases} \tag{60}$$

where $f_{i,1}\left(\varpi_{i,1}\right) = 0$, $h_{i,1} = 0$, $f_{i,2}\left(\bar{\boldsymbol{\varpi}}_{i,2}\right) = \frac{\mathrm{k}r^2}{4L_i}\sin\left(\varpi_{i,1}\varpi_{i,2}\right)$. The time delay is given by $\tau_{i,2} = 0.3s$. The virtual leader's reference signal is set as follow $y_d = \sin(t)$, in which θ_i is pendulum angle (0 is upright), $k = 100N/m$ denotes the stiffness coefficient of the connecting spring, $r = 0.5$ represents the pendulum's height, $g = 9.81m/s^2$ denotes the acceleration due to gravity and $d = 0.5$ is the spacing between the pendulum attachment. $m_1 = 2.2$, $m_2 = 2.5$, $m_3 = 3$ and $m_4 = 2.5$ are the masses at the ends of the pendulums, $L_1 = L_2 = L_3 = L_4 = 1$ are the corresponding moments of inertia.

Furthermore, the parameters for the prescribed performance and state constraints are chosen as follows, respectively: $\psi_0 = 4$, $\psi_\infty = 0.5$, $\upsilon = 1$, $l_m = 1$, $l_M = 1.5$, $\kappa_1 = 1.3 + 0.1\sin(t)$, $\kappa_2 = -1.3 + 0.1\sin(t)$. The following values are assigned to the controller

Fig. 7 Inter-execution interval in Example 1

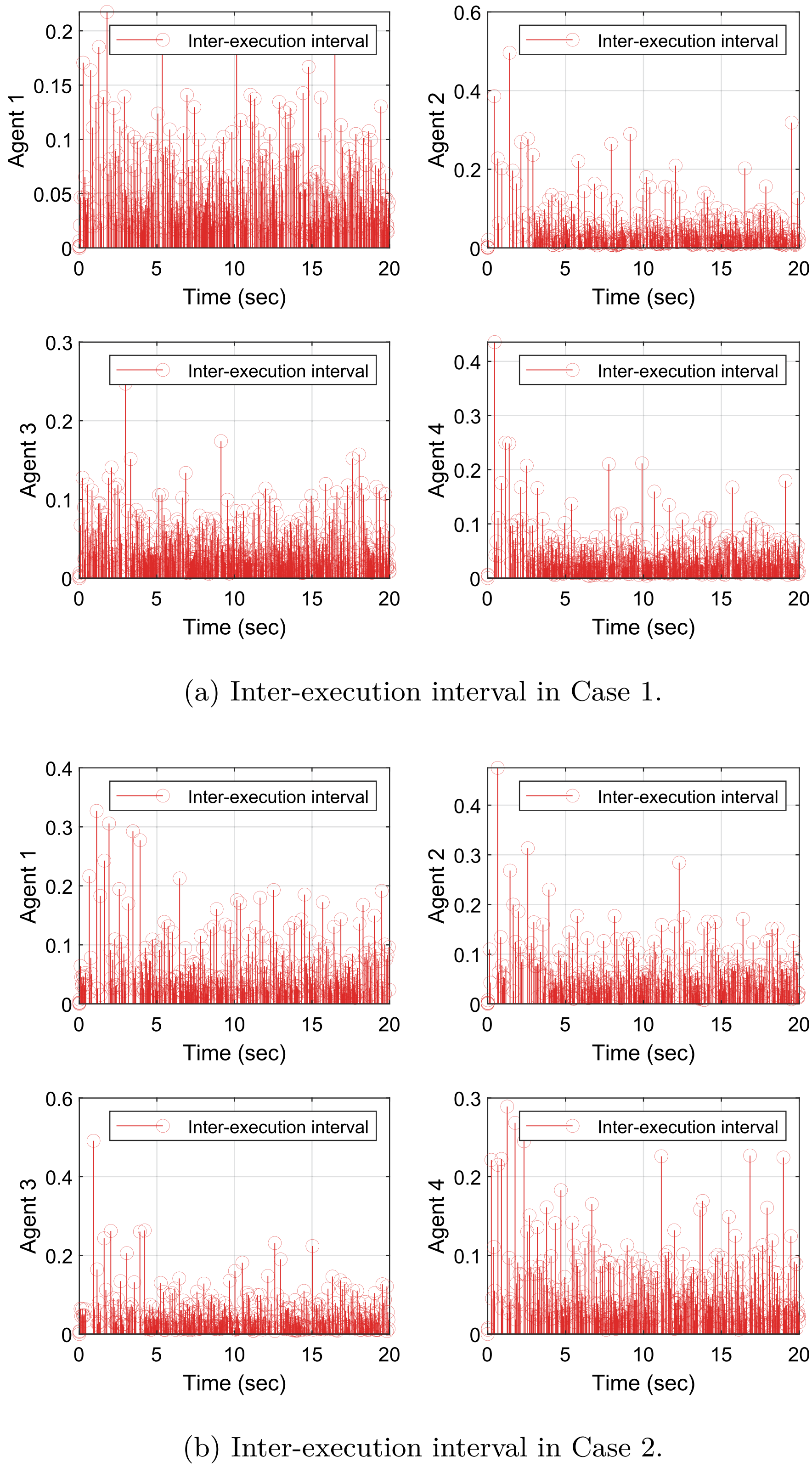


Table 1 Triggering counts and law for four follower agents

Agent Identifier	1	2	3	4
Number of Samples		3558		
Triggering Count	365	385	426	419
Triggering law (%)	10.3	10.8	12	11.8

Table 2 Triggering law comparison of two ETC schemes

Agent Identifier	1	2	3	4
Proposed ETC scheme (%)	10.3	10.8	12	11.8
Fixed ETC scheme (%)	19.5	17.5	19.9	22.7

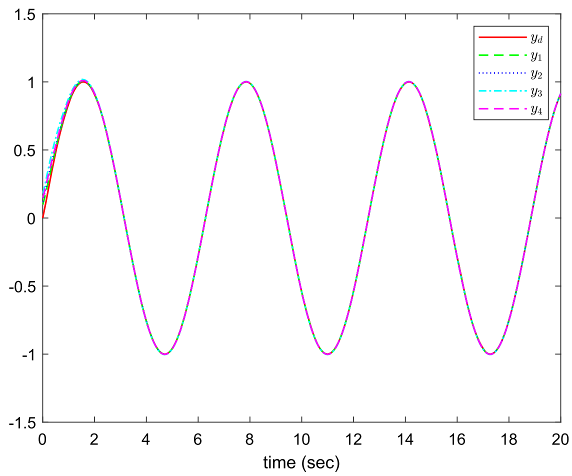


Fig. 8 System output y_i and reference signal y_d in Example 2

parameters: $k_{1,1} = k_{3,1} = 25$, $k_{2,1} = k_{4,1} = 55$, $\eta_{i,1} = 5$, $\gamma_{11} = \gamma_{4,1} = 100$, $\gamma_{2,1} = \gamma_{3,1} = 1000$, $k_{b1} = 5$, $k_{b2} = 4$, $\gamma_{1,2} = 0.1$, $\gamma_{2,2} = \gamma_{3,2} = \gamma_{4,2} = 1$, $k_{1,2} = 200$, $k_{2,2} = 500$, $k_{3,2} = k_{4,2} = 100$, $\eta_{i,2} = 2$, $rr_i = 0.5$, $\chi_i = 1$, $\chi_i' = 1$, $\varsigma_1 = \varsigma_2 = \varsigma_3 = \varsigma_4 = 1.5$, $M_1 = 0.5$, $M_2 = 0.4$, $M_3 = 0.6$, $M_4 = 0.3$, $\bar{\boldsymbol{x}}_{1,2}(0) = [0, 0]^T$, $\bar{\boldsymbol{x}}_{2,2}(0) = [0, 0]^T$, $\bar{\boldsymbol{x}}_{3,2}(0) = [0, 0]^T$, $\bar{\boldsymbol{x}}_{4,2}(0) = [0, 0]^T$.

The results from the simulation are depicted in Figs. 8, 9, 10, 11, 12. From the tracking performance in Fig. 8 and the tracking errors in Fig. 9, it can be observed that the four agents are able to track the desired signal y_d within a short period of time, and the output errors e_i satisfy the prescribed performance conditions. As shown in the system state trajectories in Fig. 10, all agent states $x_{i,2}$ remain within the predefined constraint bounds. Fig. 11 illustrates the control

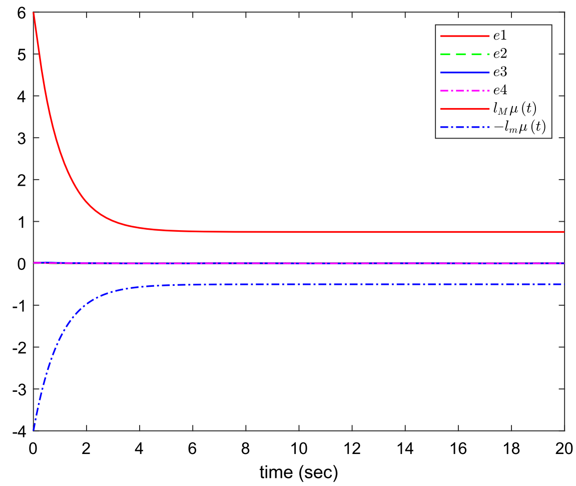


Fig. 9 Tracking error e_i in Example 2

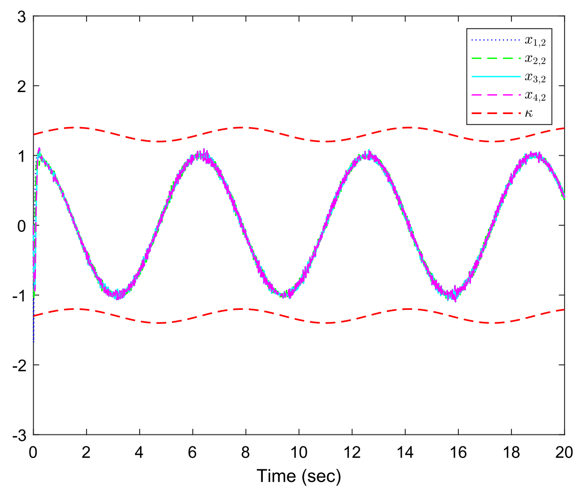


Fig. 10 System state $x_{i,2}$ in Example 2

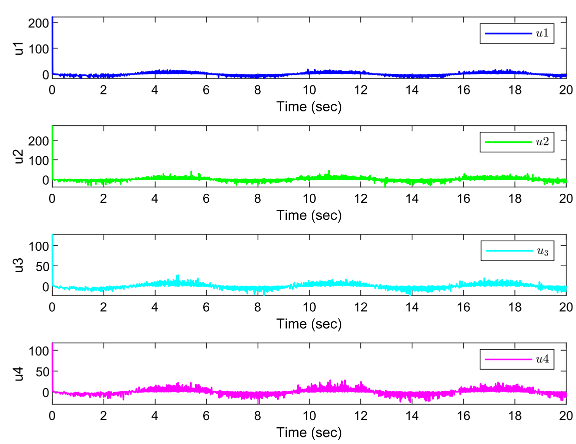


Fig. 11 Control input u_i in Example 2

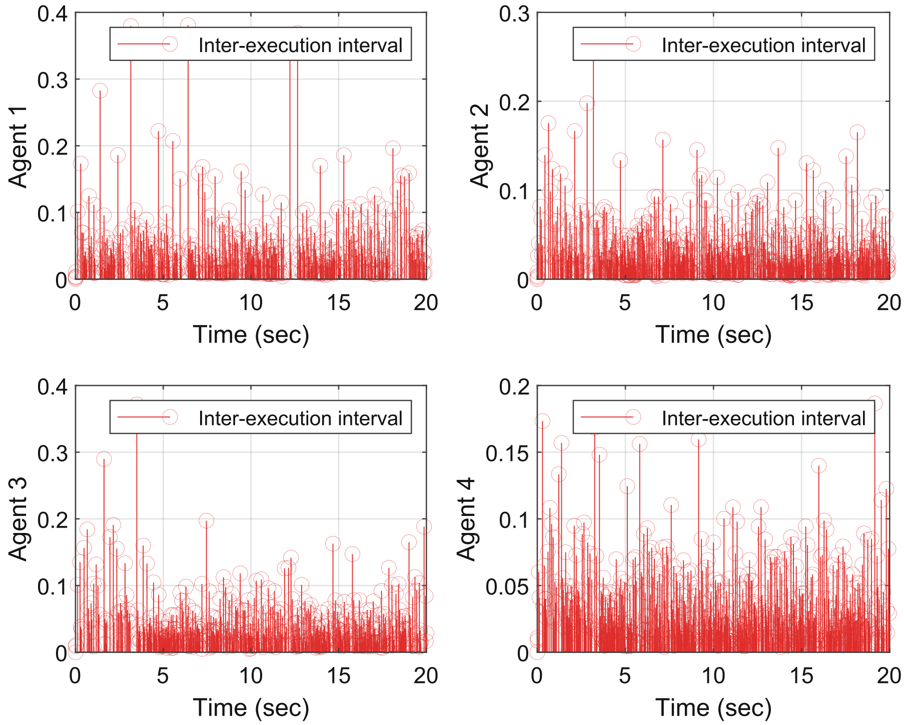


Fig. 12 Inter-execution interval in Example 2

inputs u_i of four agents. Fig. 12 displays the time intervals between two consecutive triggering events under the event-triggered mechanism, indicating that Zeno behavior does not occur.

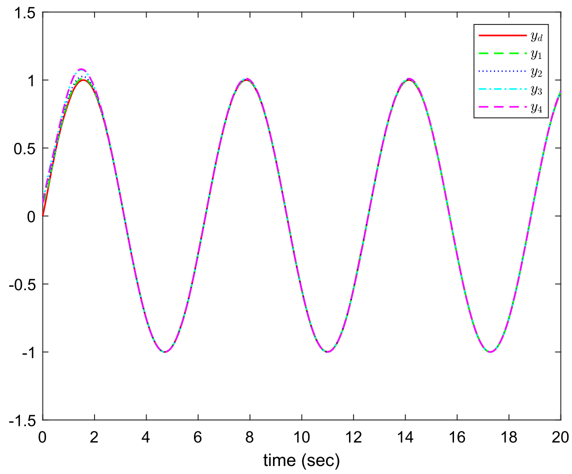
Example 3 To gain deeper insight into the performance of the proposed approach compared with existing methods, a simulation study was conducted to compare the MTN-based and RBFNN-based control schemes. The results of this comparison are shown in Fig. 13.

Fig. 13a presents the tracking performance of the four follower agents with respect to the virtual leader's reference signal under the MTN-based control scheme, while Fig. 13b shows the corresponding performance under the RBFNN-based control scheme. As illustrated in Fig. 13, both controllers achieve satisfactory tracking accuracy. Notably, the control strategy proposed in this study attains performance comparable to the RBFNN-based approach while significantly reducing computational complexity, thereby alleviating the overall computational burden.

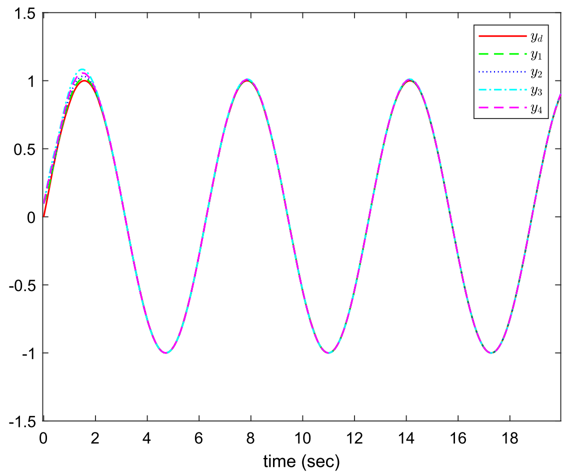
6 Conclusion

This paper investigates the cooperative control problem of MASs subject to prescribed performance, state time-delays, full state constraints and unknown external disturbances. To address these challenges, a novel ETC strategy based on MTN is proposed. Firstly, the performance function is employed to guarantee that the output error meet the prescribed performance criteria. Secondly, by combining BLFs with LK functions, the Lyapunov functions are constructed to ensure that system states satisfy the constraints while mitigating the adverse effects caused by state time-delays. At the same time, MTNs are employed to approximate the nonlinear functions encountered in the controller design. Thirdly, an ETC scheme is designed to reduce unnecessary resource consumption. Finally, the proposed control approach is validated through numerical simulation, practical example and comparative experiment.

In future work, we will comprehensively consider the impact of hybrid time-delays (state time-delays and communication delays) on systems in practical engineering applications, further enhance the engineering practicality of the proposed scheme, and conduct in-depth research on dynamic event-triggered mech-



(a) Tracking performance of MTN-based control scheme.



(b) Tracking performance of RBFNN-based control scheme.

Fig. 13 Comparison of the MTN- and RBFNN-based tracking performance

anisms to more effectively conserve communication resources.

Author contributions Zhao-Yi Zong and Yu-Qun Han wrote the main manuscript text, Fen-Fen Guan and Shan-Liang Zhu modified the main manuscript text, Zhao-Yi Zong and Yu-Qun Han prepared Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. All authors reviewed the manuscript.

Funding This work was supported by the Shandong Provincial Natural Science Foundation, China (No. ZR2020QF055).

Data Availability Statement Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

Appendix

The detailed derivation for Step p $(3 \leq p \leq n-1)$ of the controller design is presented as follows.

Step p $(3 \leq p \leq n-1)$: Design the candidate Lyapunov function $\Xi_{i,p}$ as follows

$$\begin{aligned}\Xi_{i,p} =& \Xi_{i,p-1} + \frac{1}{2}\ln\frac{k_{bp}^2}{k_{bp}^2 - z_{i,p}^2} + \frac{1}{2}\tilde{\boldsymbol{W}}_{i,p}^T\tilde{\boldsymbol{W}}_{i,p} \\ &+ \frac{1}{2}\sum_{k=1}^{p}\sum_{l=1}^{k}\int_{t-\tau_{i,k}}^{t}\varphi_{i,k,l}^2\left(x_{i,l}\left(s\right)\right)ds \\ &+ \frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{p}\sum_{l=1}^{k}\int_{t-\tau_{j,k}}^{t}\varphi_{j,k,l}^2\left(x_{j,l}\left(s\right)\right)ds \end{aligned} \tag{61}$$

where $\tilde{\boldsymbol{W}}_{i,p} = \boldsymbol{W}_{i,p} - \hat{\boldsymbol{W}}_{i,p}$ is estimation error, $\hat{\boldsymbol{W}}_{i,p}$ is the estimate of $\boldsymbol{W}_{i,p}$ and $k_{bp} = \kappa_{i,p} - \iota_{i,p}$ with $\iota_{i,p}$ is a constant.

Calculating the derivative of $\Xi_{i,p}$, and taking (1) and (5) into account, we obtain

$$\begin{aligned}\dot{\Xi}_{i,p} =& \dot{\Xi}_{i,p-1} - \tilde{\boldsymbol{W}}_{i,p}^T\dot{\hat{\boldsymbol{W}}}_{i,p} \\ &+ \frac{1}{2}\sum_{k=1}^{p}\sum_{l=1}^{k}\left[\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right) - \varphi_{i,k,l}^2\left(x_{i,l}\left(t_{i,k}^{\tau}\right)\right)\right] \\ &+ \frac{z_{i,p}}{\left(k_{bp}^2 - z_{i,p}^2\right)}\left(x_{i,p+1} + f_{i,p} + \phi_{i,p}\left(\bar{\boldsymbol{x}}_{i,p}\left(t_{i,p}^{\tau}\right)\right)\right. \\ &\left. + \Delta_{i,p} - \dot{\alpha}_{i,p-1} - \frac{z_{i,p}\dot{k}_{bp}}{k_{bp}}\right) \\ &+ \frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{p}\sum_{l=1}^{k}\left[\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right) - \varphi_{j,k,l}^2\left(x_{j,l}\left(t_{j,k}^{\tau}\right)\right)\right] \end{aligned} \tag{62}$$

where $\dot{\alpha}_{i,p-1} = \sum_{k=1}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial x_{i,k}}\left(x_{i,k+1} + f_{i,k} + \phi_{i,k}\left(\bar{\boldsymbol{x}}_{i,k}\left(t_{i,k}^{\tau}\right)\right) + \Delta_{i,k}\right) + \sum_{k=1}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial\hat{\boldsymbol{W}}_{i,k}}\dot{\hat{\boldsymbol{W}}}_{i,k} + \sum_{k=0}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial y_0^{(k)}}y_0^{(k+1)} +$

$\sum_{k=1}^{p}\sum_{j\in N_i}\frac{\partial\alpha_{i,p-1}}{\partial x_{j,k}}\left(x_{j,k+1}+f_{j,k}+\phi_{j,k}\right.$
$\left.\left(\bar{\boldsymbol{x}}_{j,k}\left(t^{\tau}_{j,k}\right)\right)+\Delta_{j,k}\right)$.

Taking $x_{i,\mathrm{p}+1}=z_{i,\mathrm{p}+1}+\alpha_{i,p}$ into consideration, and substituting (35) into (62) , while applying Lemma 5, Assumptions 3 and Assumptions 4 gives

$$
\begin{aligned}
\dot{\Xi}_{i,p}\leq&-\sum_{l=1}^{p-1}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{z_{i,p}}{\left(k_{bp}^2-z_{i,p}^2\right)}\\
&\left(z_{i,\mathrm{p}+1}+\alpha_{i,p}+F_{i,p}\right)\\
&-\frac{z_{i,p}^2\dot{k}_{bp}}{k_{bp}\left(k_{bp}^2-z_{i,p}^2\right)}+\sum_{l=1}^{p-1}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}\\
&-\tilde{\boldsymbol{W}}_{i,p}^T\dot{\hat{\boldsymbol{W}}}_{i,p}-\frac{1}{2}\frac{z_{i,p}^2}{\left(k_{bp}^2-z_{i,p}^2\right)^2}\\
&+\frac{1}{2}\sum_{q=1}^{p}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}\\
&\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right)\\
&+\frac{1}{2}\sum_{l=1}^{p-1}\varepsilon_{i,l}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\sum_{q=1}^{p}\sum_{k=1}^{q}\ell_{i,k}^2\\
&+\frac{1}{2}\sum_{q=2}^{p}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2
\end{aligned}
\tag{63}
$$

where $F_{i,p}=f_{i,p}-\sum_{k=1}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial x_{i,k}}\left(x_{i,k+1}+f_{i,k}\right)-\sum_{k=1}^{p}\sum_{j\in N_i}\frac{\partial\alpha_{i,p-1}}{\partial x_{j,k}}\left(x_{j,k+1}+f_{j,k}\right)+\frac{z_{i,p}}{\left(k_{bp}^2-z_{i,p}^2\right)}\left(1+\sum_{k=1}^{p-1}\left(\frac{\partial\alpha_{i,p-1}}{\partial x_{i,k}}\right)^2+\sum_{k=1}^{p}\sum_{j\in N_i}\left(\frac{\partial\alpha_{i,p-1}}{\partial x_{j,k}}\right)^2\right)$
$-\sum_{k=1}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial\hat{\boldsymbol{W}}_{i,k}}\dot{\hat{\boldsymbol{W}}}_{i,k}-\sum_{k=0}^{p-1}\frac{\partial\alpha_{i,p-1}}{\partial y_0^{(k)}}y_0^{(k+1)}+\frac{z_{i,p}}{2\left(k_{bp}^2-z_{i,p}^2\right)}+\frac{k_{bp}^2-z_{i,p}^2}{z_{i,p}}\tanh^2\left(\frac{z_{i,p}}{\delta_{i,p}}\right)\sum_{k=1}^{p}\sum_{l=1}^{k}\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)\right.$
$\left.+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right)$.

According to Lemma 6, we obtain

$$
\begin{aligned}
\dot{\Xi}_{i,p}=&-\sum_{l=1}^{p-1}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{z_{i,p}}{\left(k_{bp}^2-z_{i,p}^2\right)}\\
&\left(z_{i,\mathrm{p}+1}+\alpha_{i,p}+\boldsymbol{W}_{i,p}^T\boldsymbol{P}_{m_{i,p}}+E_{i,p}\right)\\
&-\frac{z_{i,p}^2\dot{k}_{bp}}{k_{bp}\left(k_{bp}^2-z_{i,p}^2\right)}-\frac{1}{2}\frac{z_{i,p}^2}{\left(k_{bp}^2-z_{i,p}^2\right)^2}\\
&+\sum_{l=1}^{p-1}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}-\tilde{\boldsymbol{W}}_{i,p}^T\dot{\hat{\boldsymbol{W}}}_{i,p}\\
&+\frac{1}{2}\sum_{q=1}^{p}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}\\
&\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right)\\
&+\frac{1}{2}\sum_{l=1}^{p-1}\varepsilon_{i,l}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2+\frac{1}{2}\sum_{q=1}^{p}\sum_{k=1}^{q}\ell_{i,k}^2\\
&+\frac{1}{2}\sum_{q=2}^{p}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2
\end{aligned}
\tag{64}
$$

Accordingly, the virtual controller $\alpha_{i,p}$ and adaptive law $\dot{\hat{\boldsymbol{W}}}_{i,p}$ are designed as follows

$$
\alpha_{i,p}=-\hat{\boldsymbol{W}}_{i,p}^T\boldsymbol{P}_{m_{i,p}}-k_{i,p}z_{i,p}-\bar{\eta}_{i,p}z_{i,p}\tag{65}
$$

$$
\dot{\hat{\boldsymbol{W}}}_{i,p}=\frac{z_{i,p}}{\left(k_{bp}^2-z_{i,p}^2\right)}\boldsymbol{P}_{m_{i,p}}-\gamma_{i,p}\hat{\boldsymbol{W}}_{i,p}\tag{66}
$$

where constant $k_{i,p}>0$, $\gamma_{i,p}>0$, $\bar{\eta}_{i,p}=\sqrt{\left(\frac{\dot{k}_{bp}}{k_{bp}}\right)^2+\eta_{i,p}}$. Obviously, $\bar{\eta}_{i,p}+\frac{\dot{k}_{bp}}{k_{bp}}\geq 0$, therefore, $-\frac{z_{i,p}^2}{k_{bp}^2-z_{i,p}^2}\left(\bar{\eta}_{i,p}+\frac{\dot{k}_{bp}}{k_{bp}}\right)\leq 0$.

Substituting (65) and (66) into (64), we have

$$
\begin{aligned}
\dot{\Xi}_{i,p}\leq&-\sum_{l=1}^{p}\frac{k_{i,l}z_{i,l}^2}{k_{bl}^2-z_{i,l}^2}+\frac{z_{i,p}z_{i,p+1}}{k_{bp}^2-z_{i,p}^2}+\sum_{l=1}^{p}\gamma_{i,l}\tilde{\boldsymbol{W}}_{i,l}^T\hat{\boldsymbol{W}}_{i,l}\\
&\frac{1}{2}\sum_{q=1}^{p}\left(1-2\tanh^2\left(\frac{z_{i,q}}{\delta_{i,q}}\right)\right)\sum_{k=1}^{q}\sum_{l=1}^{k}
\end{aligned}
$$

$$\left(\varphi_{i,k,l}^2\left(x_{i,l}\left(t\right)\right)+\sum_{j\in N_i}\varphi_{j,k,l}^2\left(x_{j,l}\left(t\right)\right)\right)$$
$$+\frac{1}{2}\sum_{l=1}^{p}\varepsilon_{i,l}^2+\frac{1}{2}\sum_{q=1}^{p}\sum_{k=1}^{q}\ell_{i,k}^2+\frac{1}{2}\sum_{j\in N_i}a_{ij}\ell_{j,1}^2$$
$$+\frac{1}{2}\sum_{q=2}^{p}\sum_{k=1}^{q}\sum_{j\in N_i}\ell_{j,k}^2 \tag{67}$$

References

1. Gao, J.K., Li, Y., Wang, B., Wu, H.B.: Multi-microgrid collaborative optimization scheduling using an improved multi-agent soft actor-critic algorithm. Energies **16**(7), 3248 (2023). https://doi.org/10.3390/en16073248
2. Garcia, G., Eskandarian, A., Fabregas, E., Vargas, H., Farias, G.: Cooperative formation control of a multi-agent Khepera iv mobile robots system using deep reinforcement learning. Appl. Sci. **15**(4), 1777 (2025). https://doi.org/10.3390/app15041777
3. Gao, S., Wen, G.G., Zhai, X.Q., Zheng, P.: Finite-/fixed-time bipartite consensus for first-order multi-agent systems via impulsive control. Appl. Math. Comput. **442**, 127740 (2023). https://doi.org/10.1016/j.amc.2022.127740
4. Li, Y.J., Tan, C.: A survey of the consensus for multi-agent systems. Syst. Sci. Control Eng. **7**, 468–482 (2019). https://doi.org/10.1080/21642583.2019.1695689
5. Wang, Y.J., Song, Y.D.: Leader-following control of high-order multi-agent systems under directed graphs: pre-specified finite time approach. Automatica **87**, 113–120 (2018). https://doi.org/10.1016/j.automatica.2017.09.017
6. Ni, J.K., Wen, C.Y., Zhao, Y.: Fixed-time leader-follower quantized output consensus of high-order multi-agent systems over digraph. Inf. Sci. **587**, 408–434 (2022). https://doi.org/10.1016/j.ins.2021.12.054
7. Shahvali, M., Azarbahram, A., Pariz, N.: Adaptive output consensus of nonlinear fractional-order multi-agent systems: a fractional-order backstepping approach. Int. J. Gen Syst **52**(2), 147–168 (2023). https://doi.org/10.1080/03081079.2022.2132488
8. Wu, L.B., Park, J.H., Xie, X.P., Ren, Y.W., Yang, Z.C.: Distributed adaptive neural network consensus for a class of uncertain nonaffine nonlinear multi-agent systems. Nonlinear Dyn. **100**, 1243–1255 (2020). https://doi.org/10.1007/s11071-020-05599-2
9. Liu, Z.J., Lu, Z.Q., Zhao, Z.J., Efe, M.Ö., Hong, K.S.: Single parameter adaptive neural network control for multi-agent deployment with prescribed tracking performance. Automatica **156**, 111207 (2023). https://doi.org/10.1016/j.automatica.2023.111207
10. Yang, T.T., Dong, J.X.: Practically predefined-time leader-following funnel control for nonlinear multi-agent systems with fuzzy dead-zone. IEEE Trans. Autom. Sci. Eng. **21**(3), 3886–3895 (2024). https://doi.org/10.1109/TASE.2023.3289445
11. Wu, W., Tong, S.C.: Fuzzy adaptive consensus control for nonlinear multiagent systems with intermittent actuator faults. IEEE Trans. Cybernetics **53**(5), 2969–2979 (2023). https://doi.org/10.1109/TCYB.2021.3123788
12. Li, K.W., Song, L.L., Li, Y.M.: Fuzzy adaptive resilient output control for nonlinear multi-agent systems under byzantine agents. IEEE Trans. Autom. Sci. Eng. **22**, 8783–8793 (2025). https://doi.org/10.1109/TASE.2024.3488820
13. Lu, L.T., Zhu, S.L., Wang, D.M., Han, Y.Q.: Predefined-time adaptive consensus control for nonlinear multi-agent systems with input quantization and actuator faults. Nonlinear Dyn. **112**(16), 14215–14234 (2024). https://doi.org/10.1007/s11071-024-09818-y
14. Lu, L.T., Zhu, S.L., Wang, D.M., Han, Y.Q.: Distributed adaptive fault-tolerant control with prescribed performance for nonlinear multiagent systems. Commun. Nonlinear Sci. Numer. Simul. **138**, 108222 (2024). https://doi.org/10.1016/j.cnsns.2024.108222
15. Hao, W.J., Sun, J.J., Zong, Z.Y., Zhu, S.L., Han, Y.Q.: Adaptive predefined time tracking control for nonlinear multi-agent systems subject to input saturation: an improved command filtering approach. Nonlinear Dyn. **113**, 19697–19716 (2025). https://doi.org/10.1007/s11071-025-11161-9
16. Yang, X.Y., Hu, J., Tan, C., Yi, X.J.: Prediction-based scaled consensus control for heterogeneous multi-agent systems under switching topologies. Nonlinear Dyn. **112**(6), 4529–4550 (2024). https://doi.org/10.1007/s11071-024-09276-6
17. Xu, Z.Q., Li, Y., Zhan, X.S., Yan, H.C., Han, Y.Y.: Time-varying formation of uncertain nonlinear multi-agent systems via adaptive feedback control approach with event-triggered impulsive estimator. Appl. Math. Comput. **475**, 128707 (2024). https://doi.org/10.1016/j.amc.2024.128707
18. Huang, R., Ding, Z.T.: Adaptive delay compensation for consensus control under switching topologies by observer-predictor. Automatica **132**, 109811 (2021). https://doi.org/10.1016/j.automatica.2021.109811
19. Liu, L., Li, X.L., Wang, H.Q., Niu, B.: Global asymptotic stabilization of stochastic feedforward nonlinear systems with input time-delay. Nonlinear Dyn. **83**, 1503–1510 (2016). https://doi.org/10.1007/s11071-015-2424-3
20. Chen, K.R., Wang, J.W., Zhang, Y., Liu, Z.: Leader-following consensus for a class of nonlinear strick-feedback multiagent systems with state time-delays. IEEE Trans. Syst. Man Cybernetics Syst. **50**(7), 2351–2361 (2020). https://doi.org/10.1109/TSMC.2018.2813399
21. Zheng, Y.L., Yang, C.D., Li, Z.X., Zhang, A.C., Qiu, J.L.: Boundary containment control of multi-agent systems with time-invariant and time-varying delays. J. Franklin Inst. **361**(11), 106923 (2024). https://doi.org/10.1016/j.jfranklin.2024.106923
22. Elahi, A., Alfi, A., Chadli, M.: Fixed-time consensus control for uncertain heterogeneous multi-agent systems with high-order dynamics and time-varying delay under generic topologies. Math. Comput. Simul. **225**, 111–128 (2024). https://doi.org/10.1016/j.matcom.2024.05.009
23. Chen, C.L.P., Wen, G.X., Liu, Y.J., Wang, F.Y.: Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks. IEEE Trans. Neural Netw. Learn. Syst. **25**(6), 1217–1226 (2014). https://doi.org/10.1109/TNNLS.2014.2302477
24. Hu, Y.Y., Du, Z.B., Wang, Y., Lv, C.C., Qu, Z.F., Wu, Z.J.: Adaptive event-triggered fuzzy control for unreliable networked control systems with time-varying delay. Int. J.

Fuzzy Syst. **26**(5), 1448–1465 (2024). https://doi.org/10.1007/s40815-024-01679-2
25. Liu, W.H., Ma, Q., Lu, Y., Xu, S.Y.: Adaptive fixed-time event-triggered fuzzy control for time-delay nonlinear systems with disturbances and quantization. IEEE Trans. Fuzzy Syst. **31**(8), 2848–2860 (2023). https://doi.org/10.1109/TFUZZ.2023.3238410
26. Wang, J.H., Yan, Y.C., Liu, Z., Chen, C.P., Zhang, C.L., Chen, K.R.: Finite-time consensus control for multi-agent systems with full-state constraints and actuator failures. Neural Netw. **157**, 350–363 (2023). https://doi.org/10.1016/j.neunet.2022.10.028
27. Zhang, Y.H., Liang, H.J., Ma, H., Zhou, Q., Yu, Z.D.: Distributed adaptive consensus tracking control for nonlinear multi-agent systems with state constraints. Appl. Math. Comput. **326**, 16–32 (2018). https://doi.org/10.1016/j.amc.2017.12.038
28. Dan, L., Lin, Z.: Adaptive finite-time consensus tracking control for nonlinear multiagent systems in nonstrict feedback form with full-state constraints. Int. J. Adapt. Control Signal Process. **35**(7), 1417–1436 (2021). https://doi.org/10.1002/ACS.3250
29. Meng, Q.K., Shahvali, M., Vrachimis, S., Polycarpou, M.M.: Fault-tolerant safe control for water networks: a backstepping neural control barrier function approach. J. Process Control **145**, 103344 (2025). https://doi.org/10.1016/j.jprocont.2024.103344
30. Jia, F.J., Xu, S.Y., Zhang, B.Y., Zhang, Z.Q.: Fuzzy-approximation adaptive prescribed performance output regulation for uncertain nonlinear systems. IEEE Trans. Syst. Man Cybernetics: Syst. **52**(7), 4300–4310 (2022). https://doi.org/10.1109/TSMC.2021.3096033
31. Han, Y.Q., Li, N., He, W.J., Zhu, S.L.: Adaptive multi-dimensional taylor network funnel control of a class of nonlinear systems with asymmetric input saturation. Int. J. Adapt. Control Signal Process. **35**(5), 713–726 (2021). https://doi.org/10.1002/acs.3224
32. Yoshimura, T.: Prescribed performance control for mimo stochastic discrete-time nonlinear systems in a strict-feedback form using a set of noisy measurements. Int. J. Syst. Sci. **53**(4), 689–703 (2022). https://doi.org/10.1080/00207721.2021.1971322
33. Sui, S., Chen, C.L.P., Tong, S.C.: A novel adaptive nn prescribed performance control for stochastic nonlinear systems. IEEE Trans. Neural Netw. Learn. Syst. **32**(7), 3196–3205 (2021). https://doi.org/10.1109/TNNLS.2020.3010333
34. Wang, W., Liang, H.J., Pan, Y.N., Li, T.S.: Prescribed performance adaptive fuzzy containment control for nonlinear multiagent systems using disturbance observer. IEEE Trans. Cybernetics **50**(9), 3879–3891 (2020). https://doi.org/10.1109/TCYB.2020.2969499
35. Liang, H.J., Zhang, Y.H., Huang, T.W., Ma, H.: Prescribed performance cooperative control for multiagent systems with input quantization. IEEE Trans. Cybernetics **50**(5), 1810–1819 (2020). https://doi.org/10.1109/TCYB.2019.2893645
36. Hu, A.H., Cao, J.D., Hu, M.F., Guo, L.X.: Event-triggered consensus of multi-agent systems with noises. J. Franklin Inst. **352**(9), 3489–3503 (2015). https://doi.org/10.1016/j.jfranklin.2014.08.005
37. Dimarogonas, D.V., Frazzoli, E., Johansson, K.H.: Distributed event-triggered control for multi-agent systems. IEEE Trans. Autom. Control **57**(5), 1291–1297 (2012). https://doi.org/10.1109/TAC.2011.2174666
38. Wang, W., Ma, H., Basin, M.V., Liang, H.J.: Adaptive event-triggered consensus control of multi-agent systems with prescribed performance and input quantization. Int. J. Adapt. Control Signal Process. **35**(8), 1454–1477 (2021). https://doi.org/10.1002/acs.3256
39. Hou, Z.C., Zhou, Z.K., Yuan, H., Wang, W.J., Wang, J., Xu, Z.: Adaptive event-triggered consensus of multi-agent systems in sense of asymptotic convergence. Sensors **24**(2), 339 (2024). https://doi.org/10.3390/s24020339
40. Li, Q., Wei, J.Y., Yuan, J.P., Gou, Q.X., Niu, Z.Q.: Distributed event-triggered adaptive finite-time consensus control for second-order multi-agent systems with connectivity preservation. J. Franklin Inst. **358**(12), 6013–6034 (2021). https://doi.org/10.1016/j.jfranklin.2021.05.028
41. Luo, Y.P., Zhu, W.L.: Event-triggered h-infinity finite-time consensus control for nonlinear second-order multi-agent systems with disturbances. Adv. Difference Equ. **2021**(1), 315 (2021)
42. Tang, Y.T., Li, R.N., Liu, H.H., Zhu, K.: Event-triggered design for optimal output consensus of high-order multi-agent systems. Int. J. Control **98**(6), 1343–1350 (2025). https://doi.org/10.1080/00207179.2024.2395322
43. Shahvali, M., Naghibi-Sistani, M.-B., Askari, J.: Distributed adaptive dynamic event-based consensus control for nonlinear uncertain multi-agent systems. Proc. Instit. Mech. Eng. Part I: J. Syst. Control Eng. **236**(9), 1630–1648 (2022). https://doi.org/10.1177/09596518221105669
44. Hashemi, M., Shahgholian, G.: Distributed robust adaptive control of high order nonlinear multi agent systems. ISA Trans. **74**, 14–27 (2018). https://doi.org/10.1016/j.isatra.2018.01.023
45. Wang, J.H., Yan, Y.C., Liu, Z., Chen, C.L.P., Zhang, C.L., Chen, K.R.: Finite-time consensus control for multi-agent systems with full-state constraints and actuator failures. Neural Netw. **157**, 350–363 (2023). https://doi.org/10.1016/J.NEUNET.2022.10.028
46. Zhang, H.W., Lewis, F.L.: Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. Automatica **48**(7), 1432–1439 (2012). https://doi.org/10.1016/j.automatica.2012.05.008
47. Yu, J.P., Zhao, L., Yu, H.S., Lin, C.: Barrier lyapunov functions-based command filtered output feedback control for full-state constrained nonlinear systems. Automatica **105**, 71–79 (2019). https://doi.org/10.1016/j.automatica.2019.03.022
48. Yao, Y.G., Tan, J.Q., Wu, J.: Event-triggered finite-time adaptive fuzzy tracking control for stochastic nontriangular structure nonlinear systems. Int. J. Fuzzy Syst. **23**(7), 2157–2169 (2021). https://doi.org/10.1007/s40815-021-01085-y
49. Li, K., Hua, C.C., You, X., Guan, X.P.: Output feedback-based consensus control for nonlinear time delay multiagent systems. Automatica **111**, 108669 (2020). https://doi.org/10.1016/j.automatica.2019.108669
50. Hao, W.J., Zong, Z.Y., Wei, S.Z., Zhu, S.L., Han, Y.Q.: Adaptive tracking control of nonlinear multi-agent systems subject to multiple constraints via multi-dimensional taylor

network. IEEE Trans. Autom. Sci. Eng. **22**, 16913–16924 (2025). https://doi.org/10.1109/TASE.2025.3581459
51. Wei, S.Y., Li, Y.X., Ahn, C.K.: Prescribed finite-time output feedback consensus tracking for nonlinear multi-agent systems with time-varying delays. IEEE Trans. Network Sci. Eng. **11**(1), 1165–1175 (2024). https://doi.org/10.1109/TNSE.2023.3321397